MATH 1210 Assignment 3 Winter 2023 **Due date: April 3**

Attempt **all** questions and show all your work. Selected number of questions will be marked.

Attach to Honesty Declaration Form

1. For a, b, c three real numbers consider the system

$$\begin{aligned} x_0 + ax_1 + a^2x_2 + a^3x_3 &= a^4 \\ x_0 + bx_1 + b^2x_2 + b^3x_3 &= b^4 \\ x_0 + cx_1 + c^2x_2 + c^3x_3 &= c^4. \end{aligned}$$

   (a) **Without** solving the system, show that it can never have a unique solution for any value of a, b and c.

   (b) Solve the system for $(a, b, c) = (0, 1, -1)$.

2. For which values of p and q is the following matrix in reduced row echelon form?

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1-q & 1 \\ 0 & 0 & p^2 - qp - 2 \end{pmatrix}$$

3. For a real number x consider the matrix

$$A = \begin{pmatrix} 1 & x & 2 \\ x & 0 & 2 \\ 1 & 1 & 1+x \end{pmatrix}.$$

   (a) Compute the determinant of A for $x = 1$.

   (b) Find all values of x for which $|A| = 0$. *Hint: use part (a).*

4. Let $\mathbf{u}$ and $\mathbf{v}$ be two nonparallel nonzero vectors with three components. Show that the vectors $\mathbf{u}$,$\mathbf{v}$ and $\mathbf{u} \times \mathbf{v}$ are linearly independent.

5. Determine if each of the following set of vectors are linearly independent.

   (a) $\mathbf{u}_1 = \langle 1, -3, 7 \rangle$, $\mathbf{u}_2 = \langle 5, 2, 3 \rangle$, $\mathbf{u}_3 = \langle 13, -7, -1 \rangle$, $\mathbf{u}_4 = \langle 1, 1, -1 \rangle$.

   (b) $\mathbf{u}_1 = \langle 2, 0, 3 \rangle$, $\mathbf{u}_2 = \langle 1, -1, 3 \rangle$, $\mathbf{u}_3 = \langle 6, -2, 2 \rangle$.

6. (a) Are the following three vectors

$$\mathbf{u}_1 = \langle 1, 5, 2, 2 \rangle, \mathbf{u}_2 = \langle 2, 4, 1, 2 \rangle, \mathbf{u}_3 = \langle -1, 1, 0, -1 \rangle$$

   linearly dependent?

   (b) Write the vector $\mathbf{v} = \langle 3, 3, -1, 1 \rangle$ as a linear combination of $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$.
   Justify your answer.

7. For a real number t consider the matrix

$$A = \begin{pmatrix} t & 0 & 6 \\ 1 & 3 & 4 \\ 0 & 1 & -1 \end{pmatrix}.$$

   (a) Find the values of t for which A is invertible.

(b) Use any method to compute A^{-1} for $t = 1$.

8. Consider the vectors

$$\mathbf{u}_1 = \langle 1, 2, 0, 0 \rangle, \qquad \mathbf{u}_2 = \langle -2, 1, 0, -7 \rangle, \qquad \mathbf{u}_3 = \langle 13, 0, 2, 0 \rangle, \qquad \mathbf{u}_4 = \langle 1, 0, 0, 3 \rangle.$$

Suppose that the vector $\mathbf{v} = \langle 4, 0, 1, 0 \rangle$ can be written as a linear combination

$$C_1\mathbf{u}_1 + C_2\mathbf{u}_2 + C_3\mathbf{u}_3 + C_4\mathbf{u}_4 = \mathbf{v}.$$

Use Cramer's rule to find C_2. You are not allowed to use any other method.

9. Find basic solutions of the homogeneous system

$$\begin{aligned} x_1 - x_2 + 3x_3 - x_4 &= 0 \\ x_1 + x_2 + x_3 + 3x_4 &= 0 \\ 2x_1 + x_2 + 3x_3 + 4x_4 &= 0 \\ 3x_1 + x_2 + 5x_3 + 5x_4 &= 0. \end{aligned}$$