MATH 1210 Assignment 1 Winter 2024 **Due date: Feburary 5, 11:59 PM**

Attempt all questions (note that selected questions will be marked), show all your work and justify your answers (unless it is explicitly stated that you do not have to do that). Unjustified answers will receive LITTLE or NO CREDIT. Also, no partial credit will be given if you do not use the method specified in the question.

1. Use mathematical induction to prove the following statement:

$$-1 + 2 - 3 + 4 - \cdots - (2n-1) + 2n = n \text{ for all positive integers } n.$$

2. Use mathematical induction to prove the following statement:

$$2^n + 1 \text{ is divisible by 3 for all odd natural numbers } n.$$

3. Use mathematical induction to prove the following statement:

$$n! < n^{n-1} \text{ for all integers } n \geq 3.$$

4. Write the sum $59 \cdot 2 - 56 \cdot 3 + 53 \cdot 4 - 50 \cdot 5 + \cdots - 14 \cdot 17$ in sigma notation so that the summation initiates with integer 1.

5. Use the properties of sigma notation, the identity $\displaystyle\sum_{k=0}^{n} x^k = \frac{x^{n+1}-1}{x-1}$, $x \neq 1$, and the identities for $\displaystyle\sum_{k=1}^{n} k^m$, where $m = 0$ or $m = 1$ (these identities were discussed in class and also appear in the textbook), to evaluate the following sum:

$$\sum_{k=n}^{2n} (\pi^k - k - 3)$$

Do NOT simplify your answer, but explicitly state all identities that you are using.

6. Use the properties of sigma notation and the identities for $\displaystyle\sum_{k=1}^{n} k^m$, where $m = 0$, 1, 2 or 3 (these identities were discussed in class and also appear in the textbook), to evaluate the following sum:

$$\sum_{k=9}^{100} (k-9)(1-(k-10)^2)$$

Explicitly state all identities that you are using and note that you do not need to simplify your final answer to a single number: for example, expressions like $\dfrac{6 \cdot 2024 \cdot 2025}{2 \cdot 3 \cdot 5}$ do not require further simplification.

7. Write the following complex expression in Cartesian from. Simplify as much as possible.

$$\frac{\overline{2+3i} \cdot (2-i)^2}{i^{23} \cdot (1+i)}$$

8. Write the following complex expression in Cartesian from. Simplify as much as possible.

$$\frac{\left(\overline{\sqrt{6}+\sqrt{2}\,i}\right)^{10}}{(-1-i^3)^{16}}$$

9. Write the following complex expression in Cartesian from. Simplify as much as possible.

$$\frac{\overline{3e^{3\pi i/10}} \cdot (2e^{-7\pi i/5})}{\left(\sqrt{2}e^{\pi i/5}\right)^4}$$

10. Find all solutions of the equation

$$\left(z^4 - 8\sqrt{2}(1+i)\right)(z^2 + z + 1) = 0$$

Write the roots in exponential form and use principal values of their arguments.