MATH 1210 Assignment 2 Winter 2024 **Due date: February 16, 11:59 PM**

Attempt all questions (note that selected questions will be marked), show all your work and justify your answers (unless it is explicitly stated that you do not have to do that). Unjustified answers will receive LITTLE or NO CREDIT. Also, no partial credit will be given if you do not use the method specified in the question.

1. Let $P(x) = ax^4 + bx^3 + x + 1$ where a and b are real numbers. Find values of a and b such that $P(2i) = (30 + 22i) + P(i)$.

2. Consider the polynomial $P(x) = 2x^4 + 5x^3 + x^2 + 10x - 6$.
   (a) What are the possible rational roots of $P(x)$?
   (b) Show that if z is a complex number for which $|z| > \frac{13}{2}$ then z can not be a zero of $P(x)$.
   (c) What are the number of possible positive real zeros and the number of possible negative real zeros of $P(x)$?
   (d) Is $x + 3$ a factor of $P(x)$? Why?
   (e) Use your answers in parts **(a)**, **(b)** and **(c)** to find the most improved form of the list of all possible rational zeros of $P(x)$ in part **(b)**.
   (f) Find all zeros of $P(x)$.

   (Roots are -3, $1/2$, $\pm\sqrt{2}i$)

3. Consider the polynomial $P(x) = x^6 - 2x^5 + 6x^4 - 4x^3 + 5x^2 + 6x - 12$.
   (a) Use the Rational Root Theorem to find all possible rational zeros of $P(x)$.
   (b) Use Descartes' Rules of Signs to determine the number of possible positive real zeros and the number of possible negative real zeros of $P(x)$.
   (c) Use the Bounds Theorem to determine how large the absolute value(modulus) of a root of $P(x)$ may be.
   (d) If $(\sqrt{3})i$ is a complex root of $P(x)$, find all zeros of $P(x)$ and express $P(x)$ as a product of linear factors.

   (Roots are ± 1, $\pm\sqrt{3}i$, $1 \pm \sqrt{3}i$)

4. Consider the matrices $A = \begin{bmatrix} 1 & a & 0 \\ 0 & -1 & a+1 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 & 0 \\ 4 & 1 & 0 \\ b & b+1 & 2 \end{bmatrix}$ where a and b are real numbers. Also let $C = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 0 & 3 \end{bmatrix}$ and $D = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Find values of a and b such that

$$AB^T - A^2 + 2CD = \begin{bmatrix} -2 & 8 & 3 \\ 0 & 0 & 6 \\ 0 & 0 & 7 \end{bmatrix}$$

   (It is a=2 and b=1)

5. For each of the following statements, if it is true prove it, and if it is not true in general, give a counter example:
   (a) $P(x) = x^7 + 5x^5 + 3x^3 + x^2 - x + 1$ has at least 4 none real complex roots.
   (b) If $P(x) = a_9x^9 + a_8x^8 + a_7x^7 + a_6x^6 + a_5x^5 + x^3 + a_0$ such that $P(0) = 0$ and $P(-x) = P(x)$, then $P(x)$ has a root with multiplicity 3.
   (c) If x_1 and x_2 are zeroes of $P(x)$ then x_1x_2 is also a zero of $P(x)$.

(d) There exist a square matrix A such that $A^4 = I$ but $A \neq I$ where I is the identity matrix of same size of A.

(e) If A is a nonzero matrix and $AB = AC$, then always $B = C$.

(f) Consider matrices A, B, C and D such A is a 3×2 and B and C are square matrices such that A^2BCD is 3×6. Then B and C are 2×2 and D is 2×6.