UNIVERSITY OF MANITOBA
COURSE: MATH 1210
DATE & TIME: April 26, 2024, 6:00–8:00 PM
FINAL EXAMINATION
DURATION: 2 hours EXAMINER: Kopotun, Moghaddam

Academic Integrity Contract: "As members of the University Community, Students have an obligation to act with academic integrity. Any Student who engages in Academic Misconduct in relation to a University Matter will be subject to discipline." (2.4 - Student Academic Misconduct Procedure).

Signature (*In Ink*): ______________________________

I understand that cheating is a serious offence.

INSTRUCTIONS

I. No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted. Smart watches are also not permitted.

II. This exam has a title page, a blank page for rough work, a bubble sheet for answering multiple choice and "true and false" questions, and 8 long answer questions. Please check that you have all the pages. You may remove the blank page if you want, but be careful not to loosen the staple.

III. The value of each question is indicated in the left-hand margin beside the statement of the question. The total value of all questions is 105 points, but 100 will be considered 100%.

IV. **Answer all questions on the exam paper** in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the same page, but CLEARLY INDICATE that your work is continued.
Show all your work and **justify** your answers. **Unjustified answers will receive LITTLE or NO CREDIT.**

V. If the QR codes on your exam paper are deliberately defaced, your exam may not be marked.

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Multiple Choice and " True or False " Questions

This is the multiple choice part of the exam. **ANSWER MULTIPLE CHOICE QUESTIONS ON THE BUBBLE SHEET AT THE END OF THIS PAPER.** There are no part marks for this section.

[3] 1. Suppose that a **nonhomogeneous** system of linear equations has infinitely many solutions. What is the correct statement?

(A) The number of equations in this system is strictly less than the number of unknowns.
(B) The number of equations in this system is greater or equal to the number of unknowns.
(C) The number of leading 1's in the RREF of the coefficient matrix is strictly less than the number of leading 1's in the RREF of the augmented matrix of this system.
(D) The number of leading 1's in the RREF of the coefficient matrix is equal to the number of leading 1's in the RREF of the augmented matrix of this system.
(E) None of the above statements is true.

[3] 2. Suppose that a polynomial of degree 10 has only the integer coefficients. What is the correct statement?

(A) This polynomial has at least one rational root.
(B) This polynomial has at least one real root.
(C) This polynomial has exactly 10 real roots (counting multiplicities), some of which may be rational.
(D) This polynomial has exactly 10 complex roots (counting multiplicities), some of which may be rational.
(E) None of the above statements is true.

[3] 3. Suppose that A and B are matrices such that the number of rows of A equals the number of columns of B, but the number of columns of A is not equal to the number of rows of B. What is the correct statement?

(A) The product AB exists.
(B) The product AB^T exists.
(C) The product BA exists.
(D) The product BA^T exists.
(E) None of the above statements is true.

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[3] 4. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$; then the solution X of the linear system of equations ,written in the matrix form, $A^{-1}X = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ is :

(A) $\begin{bmatrix} 4 \\ 4 \\ 5 \end{bmatrix}$.

(B) $\begin{bmatrix} 4 \\ 8 \\ 3 \end{bmatrix}$.

(C) $\begin{bmatrix} 5 \\ 4 \\ 0 \end{bmatrix}$.

(D) $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.

(E) None of the above.

[3] 5. If RREF of the augmented matrix of a homogeneous linear system of questions is $\left[\begin{array}{cccc|c} 1 & 2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$, then all basic solutions are:

(A) $\begin{bmatrix} -2 \\ 0 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 0 \\ 1 \end{bmatrix}$.

(B) $\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -3 \\ 1 \end{bmatrix}$.

(C) $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$.

(D) $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$.

(E) Basic solutions are not listed above, or this system has no basic solutions.

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[1] 6. Indicate if the following statement is true or false:
A matrix can have many different row echelon forms (REFs), but its reduced row echelon form (RREF) is unique.

(A) TRUE
(B) FALSE

[1] 7. Indicate if the following statement is true or false:
Let $P(x)$ be a polynomial with some complex coefficients, and suppose that z is a complex root of $P(x)$; then $\overline{z}$ is also a root of $P(x)$. ($\overline{z}$ denotes the complex conjugate of z).

(A) TRUE
(B) FALSE

[1] 8. Indicate if the following statement is true or false:
If two planes are perpendicular, then the line of intersection of these two planes is parallel to the cross product of the normal vectors to these planes.

(A) TRUE
(B) FALSE

[1] 9. Indicate if the following statement is true or false:
If a system of linear equations has 4 equations and 5 unknowns, then Cramer's rule may be used to determine the last unknown without determining the first four of them.

(A) TRUE
(B) FALSE

[1] 10. Indicate if the following statement is true or false:
If $AX = B$ is a **nonhomogeneous** system of 5 linear equations in 5 unknowns, and $\det(A) = 0$, then this system has to have infinitely many solutions.

(A) TRUE
(B) FALSE

[1] 11. Indicate if the following statement is true or false:
The set of 3 vectors with 5 components each has to be linearly dependent.

(A) TRUE
(B) FALSE

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[1] 12. Indicate if the following statement is true or false:
The transpose of the adjoint of a square matrix A is equal to the cofactor matrix of A.

(A) TRUE
(B) FALSE

[1] 13. Indicate if the following statement is true or false:
If $T : \mathbb{R}^3 \mapsto \mathbb{R}^3$ is such that $T(\langle v_1, v_2, v_3 \rangle)) = \langle v_1 + v_2, 0, v_3 + 1 \rangle)$, then T is a linear transformation.

(A) TRUE
(B) FALSE

[1] 14. Indicate if the following statement is true or false:
If A is a 5×5 real matrix and $\lambda = 0$ is its eigenvalue, then the system of equations $A\mathbf{x} = \mathbf{0}$ only has a trivial solution (where $\mathbf{x}$ is the 5×1 matrix of unknowns, and $\mathbf{0}$ is the 5×1 zero matrix).

(A) TRUE
(B) FALSE

[1] 15. Indicate if the following statement is true or false:
If $(\lambda, \mathbf{v})$ is an eigenpair of a matrix A, then $(1 + \lambda, \mathbf{v})$ is an eigenpair of the matrix $I + A$.

(A) TRUE
(B) FALSE

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Long Answer Questions

This is the long answer questions part of the exam. **FOR EACH QUESTION, ANSWER IN THE SPACE PROVIDED BELLOW THAT QUESTION. You may continue your work on the reverse side of the same page**
Show all your work and **justify** your answers. **Unjustified answers will receive LITTLE or NO CREDIT.**

[8] 1. Let $A = \begin{bmatrix} i & 1 \\ 0 & i \end{bmatrix}$ where i is the complex number for which $i^2 = -1$. Use mathematical induction to prove that $A^n = \begin{bmatrix} i^n & n\,i^{n-1} \\ 0 & i^n \end{bmatrix}$ for all integers $n \geq 2$.

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[10] 2. Let $P(x) = x^4 + ax^3 + 2ix^2 + bx - 1$, where a and b are integers. Given that $P(-x) = P(x)$ for all x; find all zeros of $P(x)$ in Cartesian form.

Hint: A polynomial $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ is equal to 0 for all x if and only if all coefficients $a_0, a_1, \cdots, a_n$ are zero.

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[10] 3. Let $\mathbf{u} = \langle -1, 2, 0, -1 \rangle$, $\mathbf{v} = \langle 3, 1, 2, 0 \rangle$ and $\mathbf{w} = \langle 3, 7, 4, 3 \rangle$. Either express the vector $\mathbf{y} = \langle 5, 1, 2, -2 \rangle$ as a linear combination of $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$, or show that this is not possible.

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[8] 4. Use Cramer's rule to solve the following linear system of questions for y **only** without solving for any of the other variables. (No mark will be given for any other method.)

$$\begin{cases} -x + 5y - z = 0 \\ 2x - y = 6 \\ 3x + 4z = -1 \end{cases}$$

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[8] 5. Let

$$A = \begin{bmatrix} 0 & 1 & -1 \\ -2 & 0 & 7 \\ 1 & -4 & 3 \end{bmatrix}.$$

Without finding A^{-1}, use the adjoint method to find the entry in the second row and third column of A^{-1}. (You are NOT asked to find all entries of A^{-1}. No mark will be given for any other method.)

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6. Let $T : \mathbb{R}^2 \mapsto \mathbb{R}^2$ be a linear transformation such that $T(\langle 1, 2\rangle) = \langle -4, -1\rangle$ and $T(\langle 2, 1\rangle) = \langle 1, -7/2\rangle$.

[10] (a) Find the matrix associated with T.

[4] (b) Find $T(\langle 3, 3\rangle)$. (Hint: you may or may not want to use part (a).)

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7. Let $A = \begin{bmatrix} 0 & 1 & -1 \\ 0 & 3 & 0 \\ 1 & 2 & 0 \end{bmatrix}$.

[6] (a) Find all eigenvalues of A (note that some of them may be complex).

[8] (b) Find all eigenvectors of A corresponding to the real eigenvalue.

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[8] 8. Let A be a 3×3 symmetric matrix with real entries that has three eigenvalues $\lambda_1 = 0$, $\lambda_2 = 1$ and $\lambda_3 = -1$. Suppose that the eigenvectors corresponding to λ_1 are

$$t\langle 0, 2, 3\rangle, \quad t \neq 0,$$

and the eigenvectors corresponding to λ_2 are

$$s\langle -1, 3, -2\rangle, \quad s \neq 0.$$

Find **all** eigenvectors corresponding to the eigenvalue λ_3.

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BLANK PAGE FOR ROUGH WORK (this page will not be marked, but it has to be returned with your exam)

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