UNIVERSITY OF MANITOBA
COURSE: MATH 1210
DATE & TIME: February 28, 2024, 5:45–7:00 PM
Midterm EXAMINATION
DURATION: 75 minutes EXAMINER: Kopotun, Moghaddam

I understand that cheating is a serious offence:

Signature (*In Ink*): ______________________________

INSTRUCTIONS

I. No calculators, texts, notes, cell phones, pagers, translators or other aids are permitted.

II. This exam contains 5 multiple choice question, 5 long answer questions and one bonus question. You do not have to answer the bonus question. The exam has a title page, 23 pages including this cover page and one blank page for rough work and one bubble sheet for answers of multiple choice questions. Please check that you have all the pages.

III. The value of each question is indicated in the left-hand margin beside the statement of the question. The total value of all questions is 60 points plus 6 points for bonus question. 60 is considered 100%.

IV. **Answer all long answer questions on the exam paper** in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

V. Show all your work and **justify** your answers. Note that unjustified answers will receive LITTLE or NO CREDIT.

VI. Please do not call or e-mail your instructor to inquire about grades. They will be available shortly after they have been marked.

VII. If the QR codes on your exam paper are deliberately defaced, your exam may not be marked.

BACK PAGE

UNIVERSITY OF MANITOBA
COURSE: MATH 1210
DATE & TIME: February 28, 2024, 5:45–7:00 PM
Midterm EXAMINATION
DURATION: 75 minutes EXAMINER: Kopotun, Moghaddam

Multiple Choice Questions

This is the multiple choice part of the exam. **ANSWER MULTIPLE CHOICE QUESTIONS ON THE BUBBLE SHEET AT THE END OF THIS PAPER.** There are no part marks for this section.

[3] 1. Let

$$w = (-1+i)i^5,$$

and suppose that

$$z_1 = -\sqrt{2}(\cos(\pi/4) + i\sin(\pi/4)),$$
$$z_2 = \sqrt{2}(\cos(3\pi/4) + i\sin(3\pi/4)),$$
$$z_3 = \sqrt{2}(\cos(5\pi/4) + i\sin(5\pi/4)).$$

Then, the following numbers are the polar form of w:

(A) z_1 only
(B) z_2 only
(C) z_3 only
(D) z_1 and z_3 only
(E) none of z_1, z_2 and z_3

[3] 2. Suppose that $P(x) = ax^4 + x^3 + bx^2 + x + 2$, where a and b are some **complex** numbers. Find all values of a and b such that $(x+1)$ is a factor of $P(x)$, and when $P(x)$ is divided by $(x+i)$ the remainder is $2i$.

(A) $a = -1-i$ and $b = 1+i$
(B) $a = -1+i$ and $b = 1-i$
(C) $a = 1-i$ and $b = -1+i$
(D) $a = 1+i$ and $b = -1-i$
(E) correct values of a and b are not listed above

[3] 3. Find all solutions of the equation $z^2 - i^3 = 0$.

(A) $\pm\dfrac{1-i}{\sqrt{2}}$
(B) $\pm\sqrt{2}(1-i)$
(C) $e^{(\pi/4+\pi k)i}, k = 0, 1$
(D) $\cos(3\pi/4) + i\sin(3\pi/4)$ and $\cos(-3\pi/4) + i\sin(-3\pi/4)$
(E) $\cos(3\pi/2) + i\sin(3\pi/2)$ and $-\cos(-\pi/2) - i\sin(3\pi/2)$

BACK PAGE

---

[3] 4. Write the sum $-\dfrac{110}{40}+\dfrac{109}{41}-\dfrac{108}{42}+\cdots-\dfrac{100}{50}$ in sigma notation.

(A) $\displaystyle\sum_{k=2}^{12}(-1)^{k+1}\frac{112-k}{38+k}$

(B) $\displaystyle\sum_{k=50}^{60}\frac{(-1)^k(50+k)}{100-k}$

(C) $\displaystyle\sum_{k=1}^{11}(-1)^{k+1}\frac{111-k}{39+k}$

(D) $\displaystyle\sum_{k=0}^{10}(-1)^{k+1}\frac{100+k}{50+(-1)^{k+1}k}$

(E) none of the above sums represents the given sum in sigma notation

[3] 5. Suppose that $z_1=a+bi$ and $z_2=c+di$ are roots of the polynomial

$$P(x)=x^3+3x^2-2x+3,$$

where a, b, c and d are real numbers such that $b\neq 0$ and $d\neq 0$. Pick the statement that is WRONG.

(A) at least one of the numbers $a+c$ and $b+d$ has to equal 0

(B) at least one of the numbers $a-c$ and $b-d$ has to equal 0

(C) the polynomial $P(x)$ has a real root which is less than 4

(D) the polynomial $P(x)$ has a real root which is less than -4

(E) both numbers a^2+b^2 and c^2+d^2 have to be less than 16

BACK PAGE

UNIVERSITY OF MANITOBA
COURSE: MATH 1210
DATE & TIME: February 28, 2024, 5:45–7:00 PM
Midterm EXAMINATION
DURATION: 75 minutes EXAMINER: Kopotun, Moghaddam

Long Answer Questions

This is the long answer questions part of the exam. **FOR EACH QUESTION, ANSWER IN THE SPACE PROVIDED BELLOW THAT QUESTION.**

[8] 6. Use mathematical induction to prove the following statement

$$\left(1-\frac{1}{1\cdot 2}\right)+\left(1-\frac{1}{2\cdot 3}\right)+\left(1-\frac{1}{3\cdot 4}\right)\cdots+\left(1-\frac{1}{(n-1)\cdot n}\right)=\frac{(n-1)^2}{n}$$

for all positive integers $n \geq 2$.

BACK PAGE

---

[9] 7. Write the following complex expression in Cartesian form. Simplify as much as possible.

$$\frac{i^{2024}}{\left(\cos(3\pi/4) + i\sin(\pi/4)\right)^{22}}$$

(Warning: is the denominator written in the polar form?)

BACK PAGE

---

[6] 8. Use the properties of sigma notation and the identities

$$\sum_{k=1}^{m} k = \frac{1}{2}\left[m(m+1)\right], \qquad \sum_{k=1}^{m} k^2 = \frac{1}{6}\left[m(m+1)(2m+1)\right],$$

to evaluate the following sum

$$\sum_{k=n}^{2n-1} \left(2k+2\right)^2.$$

(You do not need to simplify your answer at the end.)

BACK PAGE

---

9. Let $P(x) = 2x^4 + x^3 + 7x^2 + 4x - 4$.

[3] (a) Use the Rational Root Theorem to find a list of all possible rational roots of $P(x)$.

[3] (b) Use Descartes' Rules of Signs to determine the number of possible positive real zeros and the number of possible negative real zeros of $P(x)$.

[2] (c) Is $2x - 1$ a factor of $P(x)$? Why?

Question 9 is continued in the next page

BACK PAGE

$\Rightarrow$ Questin 9 is contnued.

[2] (d) Use your answers in parts **(b)** and **(c)** to show that $P(x)$ has no real zero in the interval $[1\,,\,4]$.

[4] (e) Find all zeros of $P(x)$.

BACK PAGE

---

[8] 10. Let $\mathbf{u} = \langle a, a+1, 1 \rangle$, $\mathbf{v} = \langle 1, 1, b \rangle$ where a and b are integers. Find all values of a and b such that $|\mathbf{v}| = \sqrt{6}$ and $\mathbf{u} + \mathbf{v}$ is perpendicular to $\mathbf{u} - \mathbf{v}$.

BACK PAGE

UNIVERSITY OF MANITOBA
COURSE: MATH 1210
DATE & TIME: February 28, 2024, 5:45–7:00 PM
Midterm EXAMINATION
DURATION: 75 minutes EXAMINER: Kopotun, Moghaddam

---

[6] 11. (**Bonus Question**) Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ -1 & -6 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 2 & 1 \end{bmatrix}$ and $X = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$. Find values of a and b satisfying in the equation

$$(AX + BX^T)^T + CC^T = I_2\,,$$

where I_2 is the identity matrix of order 2.
Note : In order to get partial mark for this bonus question, significant progress has to be made.

BACK PAGE

BLANK PAGE FOR ROUGH WORK (this page will not be marked, but it has to be returned with your exam)

BACK PAGE