UNIVERSITY OF MANITOBA

DATE: March 8, 2011 Midterm #2
COURSE: MATH 1210 TITLE PAGE
EXAMINATION: Techniques of Classical & Linear Algebra TIME: 45 minutes

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FAMILY NAME: (Print in ink, capitals) $\boxed{\text{Solutions}}$
GIVEN NAME(S): (Print in ink, capitals) ______________________

STUDENT NUMBER: ____________

LAB SECTION (A, B, C, or D): ______

SIGNATURE: (in ink) ______________________
(I understand that cheating is a serious offence. I have read the instructions below twice.)

- ❑ A01 R. Thomas
- ❑ A02 T. Mohammed

INSTRUCTIONS TO STUDENTS:

This is a 45 minute exam. **Please show your work clearly.**

No calculators, texts, notes, or other aids are permitted. No cellphones or electronic translators, or other electronic devices able to receive or transmit a signal are permitted.

This exam has a title page and 3 pages of questions. Please check that you have all the pages.

The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 30 points.

Answer all questions on the test paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the previous page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	9	
2	6	
3	7	
4	8	
Total:	30	

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[9] 1. Given the vectors $\mathbf{u} = \begin{bmatrix} 9 \\ 3 \\ 4 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix}$, write down

(a) $\mathbf{v} + \mathbf{u} = \begin{bmatrix} 6 \\ 4 \\ 6 \end{bmatrix}$,

(b) $7\mathbf{v} = \begin{bmatrix} -21 \\ 7 \\ 14 \end{bmatrix}$,

(c) $\|\mathbf{v}\| = \sqrt{9+1+4} = \sqrt{14}$,

(d) $\mathbf{u} \cdot \mathbf{v} = -27 + 3 + 8 = -16.$

(e) Are $\mathbf{u}$ and $\mathbf{v}$ perpendicular?

No

(f) How do you know that?

$\mathbf{u} \cdot \mathbf{v} \neq 0$

(g) What is the cosine of the angle between $\mathbf{u}$ and $\mathbf{v}$?

$\| \mathbf{u} \| = \sqrt{81 + 9 + 16} = \sqrt{106}.$

The cosine of the angle between $\mathbf{u}$ and $\mathbf{v}$ $= \dfrac{\mathbf{u} \cdot \mathbf{v}}{\| \mathbf{u} \| \| \mathbf{v} \|} = \dfrac{-16}{\sqrt{106}\sqrt{14}}.$

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[6] 2. Referring to the vectors $\mathbf{u}$ and $\mathbf{v}$ of question 1, find the point of intersection, if any, of the line $\mathbf{x} = \mathbf{u} + p\mathbf{v}$, where p is a real number, with the plane $4x + 2y + z = 30$.
Solution:

$$\mathbf{x} = \mathbf{u} + p\mathbf{v} \quad = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 3 \\ 4 \end{bmatrix} + p\begin{bmatrix} -3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 9-3p \\ 3+p \\ 4+2p \end{bmatrix}.$$

Substitute the values of x, y, and z in the plane's equation.

$$4(9-3p) + 2(3+p) + 4 + 2p = 30 \quad \Rightarrow \quad -8p = -16 \quad \Rightarrow \quad p = 2.$$

The point is $(9-3(2),\ 3+2,\ 4+2(2)) = (3,\ 5,\ 8)$.

[7] 3. Solve for the unknown matrix X the matrix equation $A + B^\top - X = C + 3I$, given that $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -1 \\ 1 & 2 \end{bmatrix}$, and $C = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$,
Solution:

$$X = A + B^T - C - 3I = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & 4 \end{bmatrix}.$$

[8] 4. Given the equation $x^2 + 16x - 17 = 0$, state in specific detail what each of

(a) the fundamental theorem of algebra,

The equation has two roots counting multiplicity.

(b) the rational-roots theorem,

Rational roots can be only $\pm 1,\ \pm 17$.

(c) the bounds theorem, and

Let $M = \max\{16,\, 17\} = 17$, then for any root x

$$|\,x\,| < \frac{M}{a_n} + 1 = \frac{17}{1} + 1 = 18.$$

(d) Descartes' rules of signs tells us about the equation.

There is one positive root.
If $P(x) = x^2 + 16x - 17$, then $P(-x) = x^2 - 16x - 17$.
So there is one negative root.

(e) Solve the equation without using the quadratic-equation formula.

$x^2 + 16x - 17 = (x - 1)\,(x + 17) = 0$. Roots are 1 and -17.