Last Name (Print)______________________________

First Name (Print)______________________________

I understand that cheating is a serious offense.

Signature:______________________________

Student Number______________________________

Room ______________ Seat Number______________

THE UNIVERSITY OF MANITOBA
DEPARTMENT OF MATHEMATICS

MATH 1300 Vector Geometry and Linear Algebra Mid-Term Exam

Date: Thursday, February 22, 2007
Time: 5:30–6:30 PM

Identify your section by marking an X in the box.

	Section	Instructor	Slot	Time	Room
☐	A01	E. Schippers	5	TTh 10:00–11:15am	208 Armes
☐	A02	N. Zorboska	8	MWF 1:30–2:20pm	204 Armes
☐	A03	D. Kelly	12	MWF 3:30–4:20pm	208 Armes
☐	A04	C. Platt	15	TTh 4:00–5:15pm	200 Armes
☐	A05	J. Sichler	E2	T 7:00–10:00pm	204 Armes
☐	Other	(challenge, deferred, etc.)			

DO NOT WRITE IN THIS COLUMN

1	/9
2	/8
3	/9
4	/9
5	/9
6	/8
7	/8
Total	/60

Instructions

*Fill in **all** the information above.*

This is a one-hour exam.

***No** calculators, texts, notes, or other aids are permitted.*

***Show your work clearly** for full marks.*

*This exam has 7 questions on 4 numbered pages, for a total of 60 points. **Check now** that you have a complete exam.*

*Answer all questions on the exam paper in the space provided. If you need more room, you may continue your answer on the **reverse** side, but **clearly indicate** that your work is continued there. You may also use the backs of pages for scratch work, but none of it will be marked unless clearly indicated otherwise.*

*If a question calls for a specific method, **no credit** will be given for other methods.*

[Values]

[9] **1.** Consider the linear system:

$$\begin{array}{rcrcrcrcr} x_1 & + & 2x_2 & & & + & 5x_4 & = & 4 \\ x_1 & + & 2x_2 & + & 2x_3 & - & x_4 & = & 8 \end{array}$$

(a) Find the general solution to this system using Gauss-Jordan elimination.

(b) Find a solution to the above system with $x_2 = -2$ and $x_4 = 3$.

[8] **2.** Let $A = \begin{bmatrix} 1 & 2 \\ -4 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 0 \end{bmatrix}$, and $C = \begin{bmatrix} -1 & 2 \\ -3 & 0 \\ 0 & 5 \end{bmatrix}$.

In each part below, evaluate the expression or state that it does not exist.
If the expression does not exist, give a reason.

(a) $AB + C$

(b) $AC + B$

(c) $BC + A$

Continued ⟶

[Values]

[9] **3.** Let $A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 2 & -1 & 0 \end{bmatrix}$. Find A^{-1} by the method of row reduction. Show all your work. Write your final answer where indicated at the bottom of the page.

Answer: $A^{-1} = \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$

Continued ⟶

[Values]

[9] **4.** Express $A = \begin{bmatrix} 0 & 2 \\ 1 & -3 \end{bmatrix}$ as a product of elementary matrices. Show all your work.

[9] **5.** Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 1 \\ 3 & 1 & 0 \end{bmatrix}$, and assume B is another 3×3 matrix with $\det(B) = 10$.

(a) Find $\det(A)$ by expansion along row 2. (*No credit* for any other method.)

(b) Find the determinant of AB^2.

(c) Find the determinant of $A^{-1}(2B)A^T$.

[Values]

[8] **6.** Use Cramer's rule to solve the following system. (*No credit* for any other method.)

$$\begin{array}{rrcr} 2x & +5y & = & 6 \\ 3x & +2y & = & -7 \end{array}$$

[8] **7.** Assume that the augmented matrix of a certain linear system can be reduced to

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & p & q \end{array}\right]$$

with elementary row operations.

Determine all values of p and q (if any) for which this system

(a) has **no** solutions:

(b) has a **unique** solution:

(c) has **infinitely many** solutions:

(d) In case **(**c), determine the **general solution**.