UNIVERSITY OF MANITOBA

FAMILY NAME: (Print in ink) _____

GIVEN NAME(S): (Print in ink) _____

STUDENT NUMBER: _____

SIGNATURE: (in ink) _____
(I understand that cheating is a serious offense)

❑   A01   Dr. Michael Doob   [MWF 9:30-10:20]

❑   A02   Dr. Julien Arino   [TR 8:30-9:45]

❑   A03   Dr. Michelle Davidson   [MWF 11:30-12:20]

❑   A04   Dr. David Gunderson   [MWF 13:30-14:20]

❑   A05   Dr. Julien Arino   [TR 11:30-12:45]

❑   A91   Challenge for Credit

❑   Deferred examination

INSTRUCTIONS TO STUDENTS:

This is a 120 minute exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 11 pages of questions, and one blank page (which may be removed). Please check that you have all the pages.

The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 80 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	20	
2	8	
3	6	
4	5	
5	13	
6	9	
7	4	
8	6	
9	9	
Total:	80	

UNIVERSITY OF MANITOBA

DATE: December 18, 2009   FINAL EXAMINATION
PAPER # 507   PAGE: 1 of 11
COURSE: MATH 1300   TIME: 120 minutes
EXAMINATION: Vector Geometry and Linear Algebra   EXAMINER: Various

[3]   1.   (a) Let V be a vector space, $W \subset V$. Under what conditions is W a subspace of V?

[4]   (b) Let A be an $n \times n$-matrix. State four additional properties equivalent to "A is invertible".

[3]   (c) Prove that $u \bullet (v \times w) = v \bullet (w \times u)$.

[2]   (d) Prove the Cauchy-Schwarz inequality in 3-space.

[3]   (e) State 3 of the axioms that must be satisfied for a set V to be a vector space.

(f) Let $S = \{v_1, \ldots, v_p\}$ be p vectors in an n-dimensional vector space V. State whether the following statements are True or False.
[Note: the score on question (f) is the number of right answers **minus** the number of right answers, with a minimum of 0.]

[1]   i. If S is linearly independent, then $p = n$: _____

[1]   ii. If S is a basis of V, then $p = n$: _____

[1]   iii. If $\mathrm{Span}(S) = W$, then W is a subspace of V: _____

[1]   iv. If S is linearly independent, then S spans V: _____

[1]   v. Let $S' = \{w_1, \ldots, w_p\}$ be a basis of V. Then $S = S'$: _____

UNIVERSITY OF MANITOBA

DATE: December 18, 2009   FINAL EXAMINATION
PAPER # 507   PAGE: 3 of 11
COURSE: MATH 1300   TIME: 120 minutes
EXAMINATION: Vector Geometry and Linear Algebra   EXAMINER: Various

2. Let
$$W = \left\{ M \in \mathcal{M}_{22}; M = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}, \text{ for all } a, b, d \in \mathbb{R} \right\}.$$

[4]   (a) Show that W is a subspace of $\mathcal{M}_{22}$, the vector space of 2×2-matrices.

[4]   (b) Is
$$S = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 2 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$$
a basis of W? If not, how should S be modified in order to be a basis of W?

[6] 3. One of the matrices

$$A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

is expressible as a product of elementary matrices, the other is not. Express the one that is expressible as a product of elementary matrices as such, and explain why the other is not.

[5] 4. Is the vector $-1 + 9x - x^2$ in the span of the vectors

$$\{1 + x + 2x^3, 3x + x^2 - x^3, 1 - x + 2x^2\}?$$

Explain.

UNIVERSITY OF MANITOBA

DATE: December 18, 2009   FINAL EXAMINATION
PAPER # 507   PAGE: 5 of 11
COURSE: MATH 1300   TIME: 120 minutes
EXAMINATION: Vector Geometry and Linear Algebra   EXAMINER: Various

5. Let $P(1, 2, 3)$ and $Q(2, 3, 1)$ be two points in $\mathbb{R}^3$. The aim of this exercise is to find the coordinates of the points $R(a, b, c) \in \mathbb{R}^3$ such that

$$\overrightarrow{PQ} \perp \overrightarrow{PR} \quad \text{and} \quad \|\overrightarrow{QR}\| = 3, \tag{*}$$

where the symbol $\perp$ means "orthogonal to".

[2]   (a) Find an equation that must be satisfied by the coordinates a, b, c of R in order that $\overrightarrow{PQ} \perp \overrightarrow{PR}$.

[1]   (b) What is the nature of the set of values of a, b, c satisfying the condition found in (a)?

[2]   (c) State the Pythagorean Theorem in n-space.

[2]   (d) Compute the norms of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

[2]   (e) Using the Pythagorean Theorem, find an equation that must be satisfied by the coordinates a, b, c of R. [Hint: this equation is not linear.]

[1]   (f) What is the nature of the set of values of a, b, c satisfying the conditions found in (e)? [You can use a geometrical argument.]

[2]   (g) Deduce a system characterizing the set of points satisfying (*).

[1]   (h) What is the nature of the set of values of a, b, c satisfying the conditions found in (g)?

[3]   6.   (a) Find the point of intersection of the lines L_1 and L_2 defined by

$$L_1 \ : \ (x, y, z) = (1, 2, 1) + t(2, 1, 2), \quad t \in \mathbb{R}$$
$$L_2 \ : \ (x, y, z) = (2, 1, 2) + s(1, 2, 1), \quad s \in \mathbb{R}.$$

[3]   (b) Find an equation of the plane containing the two lines L_1 and L_2.

[3]   (c) Find the point on L_1 closest to the point $(3, 2, 0)$.

7. Let $P(1, 2, 3)$ be a point and $(x, y, z) = (4, 1, 2) + t(1, 2, -1)$, $t \in \mathbb{R}$, be the equation of a line L in $\mathbb{R}^3$.

[2]   (a) Find the equation of the plane through P perpendicular to the line L.

[2]   (b) Find the equation of a line through P perpendicular to the plane with equation $2x - y - z + 4 = 0$.

UNIVERSITY OF MANITOBA

DATE: December 18, 2009   FINAL EXAMINATION
PAPER # 507   PAGE: 9 of 11
COURSE: MATH 1300   TIME: 120 minutes
EXAMINATION: Vector Geometry and Linear Algebra   EXAMINER: Various

8. Consider the following system of linear equations:

$$
\begin{aligned}
x + y + z &= 1 \\
2x + y + z &= 2 \\
3x + ay + bz &= c.
\end{aligned}
$$

For what values of a, b and c does this system have

[2] (a) no solutions

[2] (b) one solutions

[2] (c) more than one solution

UNIVERSITY OF MANITOBA

DATE: December 18, 2009   FINAL EXAMINATION
PAPER # 507   PAGE: 10 of 11
COURSE: MATH 1300   TIME: 120 minutes
EXAMINATION: Vector Geometry and Linear Algebra   EXAMINER: Various

9. The matrix

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 & 0 \\ 1 & 1 & 2 & 1 & 2 \\ 3 & 4 & 5 & 2 & 4 \\ 1 & 3 & 0 & -1 & -2 \\ 0 & -1 & 1 & 1 & 2 \end{bmatrix}$$

has reduced row echelon form:

$$\begin{bmatrix} 1 & 0 & 3 & 2 & 4 \\ 0 & 1 & -1 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

[1]   (a) The dimension of the row space of A is _____

[2]   (b) A basis for the row space of A is:

[1]   (c) The dimension of the column space of A is _____

[2]   (d) A basis for the column space of A is:

[1]   (e) The dimension of the null space of A is _____

[2]   (f) A basis for the null space of A is:

UNIVERSITY OF MANITOBA

DATE: December 18, 2009   FINAL EXAMINATION
PAPER # 507   PAGE: 11 of 11
COURSE: MATH 1300   TIME: 120 minutes
EXAMINATION: Vector Geometry and Linear Algebra   EXAMINER: Various

BLANK PAGE FOR ROUGH WORK