UNIVERSITY OF MANITOBA

DATE: October 25th, 2010 (Monday)
PAPER # 35
DEPARTMENT & COURSE NO: MATH 1300
EXAMINATION: Vector Geometry & Linear Algebra

MIDTERM EXAMINATION

TIME: 1 hour
EXAMINER: Various

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[9] 1. Solve, by Gauss-Jordan elimination, the linear system

$$\begin{array}{rcrcrcr} x & & & + & 3z & = & 2 \\ x & + & y & + & 3z & = & 0 \\ & & y & + & z & = & -1. \end{array}$$

1

$$\left[\begin{array}{ccc|c} 1 & 0 & 3 & 2 \\ 1 & 1 & 3 & 0 \\ 0 & 1 & 1 & -1 \end{array}\right] \xrightarrow{R_2' = R_2 - R_1} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 1 & 1 & -1 \end{array}\right]$$

$$\xrightarrow{R_3' = R_3 - R_2} \left[\begin{array}{ccc|c} 1 & 0 & 3 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right] \xrightarrow{R_1' = R_1 - 3R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$\therefore\ x = -1,\quad y = -2,\quad z = 1$

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[8] 2. Suppose that the augmented matrix of a system of linear equations has been partially reduced using elementary row operations to

$$\left[\begin{array}{ccc|c} 1 & -1 & a-2 & 3 \\ 0 & 1 & 2b+1 & -1 \\ 0 & 0 & a-1 & b+3 \end{array}\right].$$

(a) Find all values (if any) of a and b for which the system is inconsistent.

$a = 1 \quad , \; b \neq -3$

(b) Find all values (if any) of a and b for which the system has a unique solution.

$a - 1 \neq 0$

(c) Find all values (if any) of a and b for which the system has infinitely many solutions and for these values of a and b find all solutions.

$a = 1 \; , \; b = -3$

$$\left.\begin{array}{l} x - y + (a-2) z = 3 \\ y + (2b+1) z = -1 \end{array}\right\} \Rightarrow \begin{array}{l} x - y - z = 3 \\ y - 5z = -1 \end{array}$$

Let, $z = t \; (t \text{ real})$

$\therefore \; y = 5t - 1 \quad , \quad x = 3 + y + z$

$= 3 + 5t - 1 + t = 6t - 2$

[9] 3. Let $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 2 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$. In each of the following cases, calculate the expression or briefly give a reason why it can not be calculated:

(a) $B + 2A^T$;

$$= \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -2 \\ 4 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -3 \\ 4 & 1 \\ 2 & 3 \end{bmatrix}$$

(b) BA;

$$= \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & -1 \\ -1 & 0 & 1 \\ 1 & 4 & 1 \end{bmatrix}$$

(c) $AB - C^2$.

$$= \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ 1 & 1 \end{bmatrix}$$

[10] 4. Given $3A^{-1} = \begin{bmatrix} 3 & 0 \\ -6 & 3 \end{bmatrix}$, and B is derived from A by adding -3 times row 2 to row 1 (*i.e.*, $R_1' = R_1 - 3R_2$),

(a) find A;

$$3A^{-1} = \begin{bmatrix} 3 & 0 \\ -6 & 3 \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}.$$

(b) find B;

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \xrightarrow{R_1' = R_1 - 3R_2} \begin{bmatrix} -5 & -3 \\ 2 & 1 \end{bmatrix} = B$$

(c) find an elementary matrix E such that $EA = B$.

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \xrightarrow{R_1' = R_1 - 3R_2} \begin{bmatrix} 1 & -3 \\ 0 & 1 \end{bmatrix} = E$$

$$\therefore EA = B.$$

[10] 5. A is a 3×3 matrix with $\det(A) = 2$. Find $\det(B)$ when B is obtained by:

(a) dividing a row of A by a nonzero scalar k;

$$\det(B) = \frac{1}{K} \det(A) = 2/K$$

(b) interchanging any two rows of A;

$$\det(B) = -\det(A) = -2$$

(c) $B = A^2 A^T$;

$$\det(B) = \det(A)\det(A)\det(A^T)$$
$$= 2 \cdot 2 \cdot 2 = 8$$

(d) $B = -2A$;

$$\det(B) = \det(-2A) = (-2)^3 \det(A)$$
$$= -8 \cdot 2 = -16$$

(e) $B = A\, adj(A)$.

$$= A A^{-1} \det(A)$$

$$\Rightarrow \det(B) = \det(A) \cdot \frac{1}{\det(A)} \cdot (2)^3 = 8.$$

[7] 6. Find the **solution for y only** of the following system by using Cramer's Rule:

$$\begin{array}{rrrrrr} 5x & +y & - & z & = & -7 \\ 2x & -y & - & 2z & = & 6 \\ 3x & & + & 2z & = & -7. \end{array}$$

$$|A| = \begin{vmatrix} 5 & 1 & -1 \\ 2 & -1 & -2 \\ 3 & 0 & 2 \end{vmatrix} = (-1)\begin{vmatrix} 2 & -2 \\ 3 & 2 \end{vmatrix} + (-1)\begin{vmatrix} 5 & -1 \\ 3 & 2 \end{vmatrix}$$

$$= -10 - 13 = -23$$

$$|A_2| = \begin{vmatrix} 5 & -7 & -1 \\ 2 & 6 & -2 \\ 3 & -7 & 2 \end{vmatrix}$$

$$= 5(12-14) + 7(4+6) + (-1)(-14-18)$$

$$= 92$$

$$\therefore y = \frac{92}{-23} = -4.$$

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[7] 7. Given any two $n \times n$ square invertible matrices A and B, and I the identity matrix of size $n \times n$, answer the following questions whether true or false.

(a) the system $A\mathbf{x} = \mathbf{b}$ can have infinitely many solutions:

True ☐ False ☑

(b) A^{-1} is a product of elementary matrices:

True ☑ False ☐

(c) A and B are row equivalent:

True ☑ False ☐

(d) if $AB = I$, then $B = A^{-1}$:

True ☑ False ☐

(e) if R is the row-reduced echelon form of A then $R = I$:

True ☑ False ☐

(f) $adjA$ is invertible:

True ☑ False ☐

(g) $det(AB) = 0$:

True ☐ False ☑