3-D numerical simulation model of biogas production for anaerobic digesters

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Objectives
(1) To conduct a comprehensive literature review on modeling biogas production from anaerobic digestion. (2) To develop a general and fundamentally-based three-dimensional numerical model that predicts biogas production from plug-flow type digesters, and (3) To validate the model predictions against experimental data obtained from the literature.

Assumptions
Biogas production in an anaerobic digester is a chemical reaction process, which is governed by mass and momentum conservations, turbulence, energy balance, species transport, and chemical reactions. This model was developed based on the following assumptions:
• Heat flow and species transport through the digester are 3-D steady.
• The model is limited to plug-flow anaerobic digesters where fluid flow is very low. The momentum equations and turbulence model are negligible.
• The digester cover, walls, and floor are adiabatic.
• The liquid manure is assumed to be Newtonian fluid.
• The manure temperature is constant at 32° before species reaction.
• Species reaction is only taken place by one step, i.e., reactants can be directly converted into products without any intermediate products.

Governing equations

Mass conservation equation:
\[
\frac{\partial}{\partial t} \rho + \frac{\partial}{\partial x_1} \left( \rho u_1 \right) + \frac{\partial}{\partial x_2} \left( \rho u_2 \right) + \frac{\partial}{\partial x_3} \left( \rho u_3 \right) = 0
\]

Energy equation:
\[
\frac{\partial}{\partial t} \left( \rho e \right) + \frac{\partial}{\partial x_1} \left( \rho e u_1 \right) + \frac{\partial}{\partial x_2} \left( \rho e u_2 \right) + \frac{\partial}{\partial x_3} \left( \rho e u_3 \right) \]
\[
= \frac{\partial}{\partial x_1} \left( \rho k \frac{\partial T}{\partial x_1} \right) + \frac{\partial}{\partial x_2} \left( \rho k \frac{\partial T}{\partial x_2} \right) + \frac{\partial}{\partial x_3} \left( \rho k \frac{\partial T}{\partial x_3} \right) + \rho q + \rho \dot{u}_i \left( \frac{\partial T}{\partial x_i} \right)
\]

Species transport equation:
\[
\frac{\partial}{\partial t} \left( \rho \xi_j \right) + \frac{\partial}{\partial x_1} \left( \rho u_1 \xi_j \right) + \frac{\partial}{\partial x_2} \left( \rho u_2 \xi_j \right) + \frac{\partial}{\partial x_3} \left( \rho u_3 \xi_j \right) = \frac{\partial}{\partial x_1} \left( \rho \frac{\partial C_j}{\partial x_1} \right) + \frac{\partial}{\partial x_2} \left( \rho \frac{\partial C_j}{\partial x_2} \right) + \frac{\partial}{\partial x_3} \left( \rho \frac{\partial C_j}{\partial x_3} \right) + \rho \dot{c}_j + \rho \dot{W}_j
\]

Chemical reaction equation:
\[
C_3H_6O_6 + H^+ + H^+ \rightarrow C_2H_2O_2 + H_2O
\]

Modeling reaction rate:
by Arrhenius expression:
\[
A_i = A_0 e^{-\frac{E_a}{RT}}
\]
and calculating rate by first-order BOD removal rate:
\[
\frac{dC_i}{dt} = -k_i C_i
\]

Modeling validation

Daily manure flow rate=38.336 m³/day
HRT=3590906 s (41 days and 13 hrs)
Retention temperature= 32°C

Conclusions
(1) A general three-dimensional numerical simulation model that predicts biogas production from plug-flow type digesters is developed. The model is based on the principles of mass conservation, energy balance, species transport, and chemical reactions.
(2) Model prediction for a plug-flow anaerobic digester is validated against experimental data obtained from the literature. The results agree within 5%.

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