Attempt all questions and **show all your work**. Due **A01**: Thu Oct 17, **A02**: Fri Oct 18.

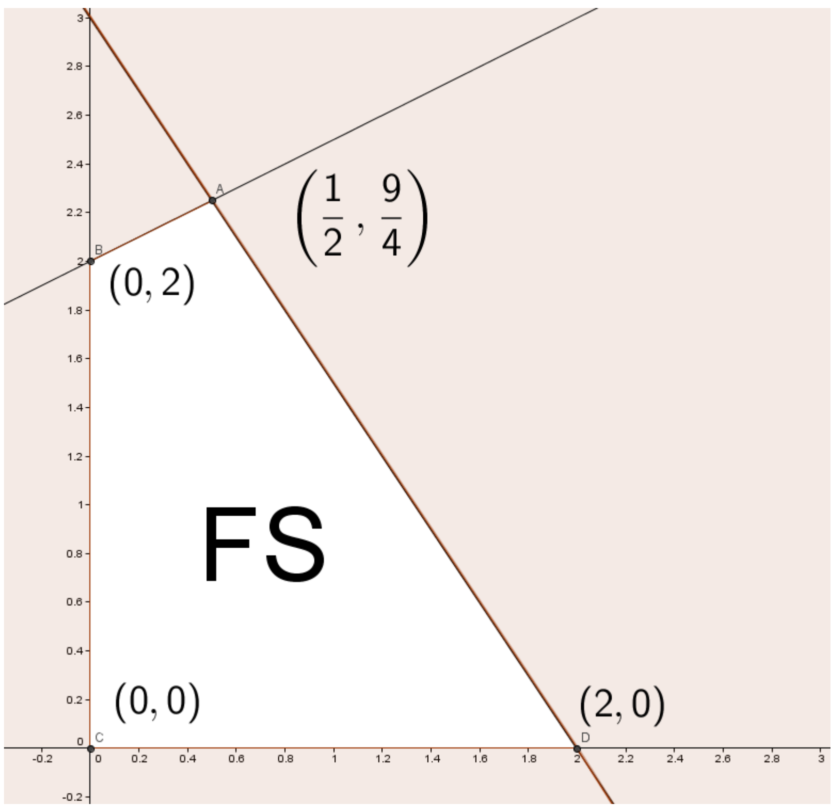
Assignments must be handed in during class time. Any assignment handed in after class is over are considered late and will not be accepted. Assignments must include a signed honesty declaration and assignments that do not do so will not be marked. The total value of all questions is 90 points.

[10] 1. Maximize $z = 2x + 5y$ subject to

$$3x + 2y \leq 6 \qquad -x + 2y \leq 4$$

$$x \geq 0 \qquad y \geq 0.$$

Solution:



Check the corner points in the objective function:

x	y	$z = 2x + 5y$
0	2	10
0	0	0
2	0	4
$\frac{1}{2}$	$\frac{9}{4}$	$\frac{49}{4} = 12.25$

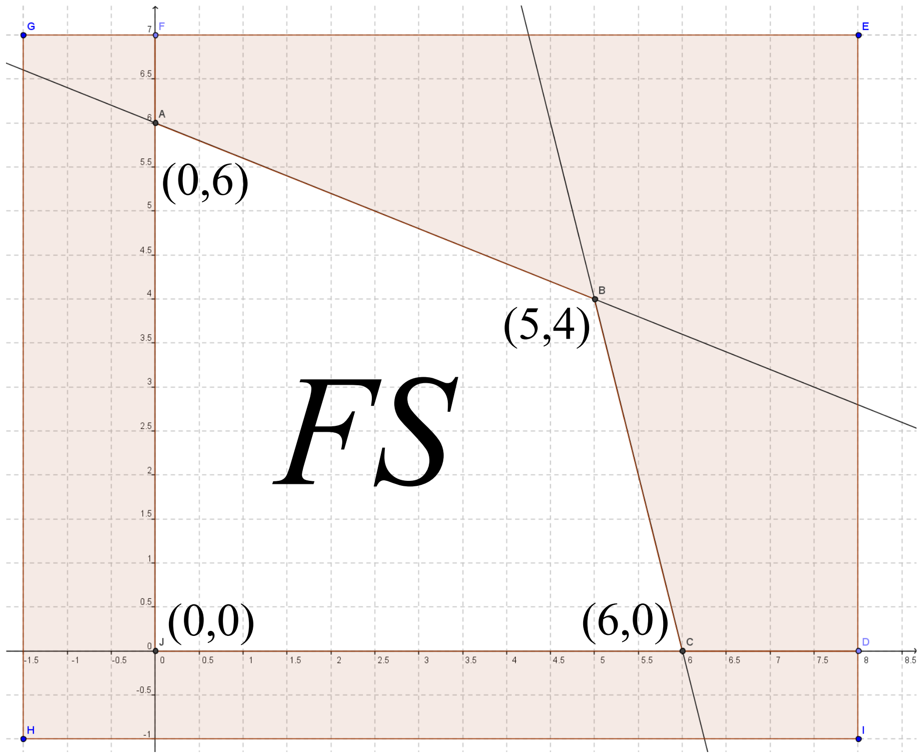
Therefore, z is maximized at the point $(\frac{1}{2}, \frac{9}{4})$.

[10] 2. Minimize $f = x + 2y$ subject to

$$y \leq \frac{-2}{5}x + 6, \qquad y \leq -4x + 24,$$

$$x \geq 0, \qquad y \geq 0.$$

Solution:



Check the corner points in the objective function:

x	y	$f = x + 2y$
0	6	12
5	4	13
6	0	6
0	0	0

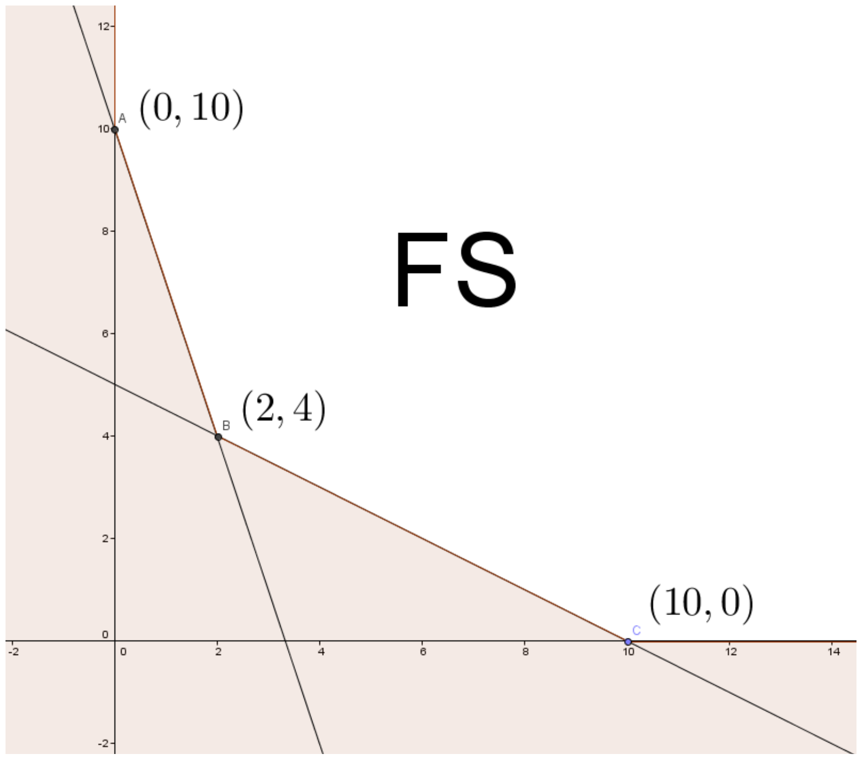
Therefore, f is minimized at $(0, 0)$.

[10] 3. Minimize $z = 2x + 4y$ subject to

$$x + 2y \geq 10 \qquad 3x + y \geq 10$$

$$x \geq 0 \qquad y \geq 0.$$

Solution:



Check the corner points in the objective function:

x	y	$z = 2x + 4y$
0	10	40
2	4	20
10	0	20

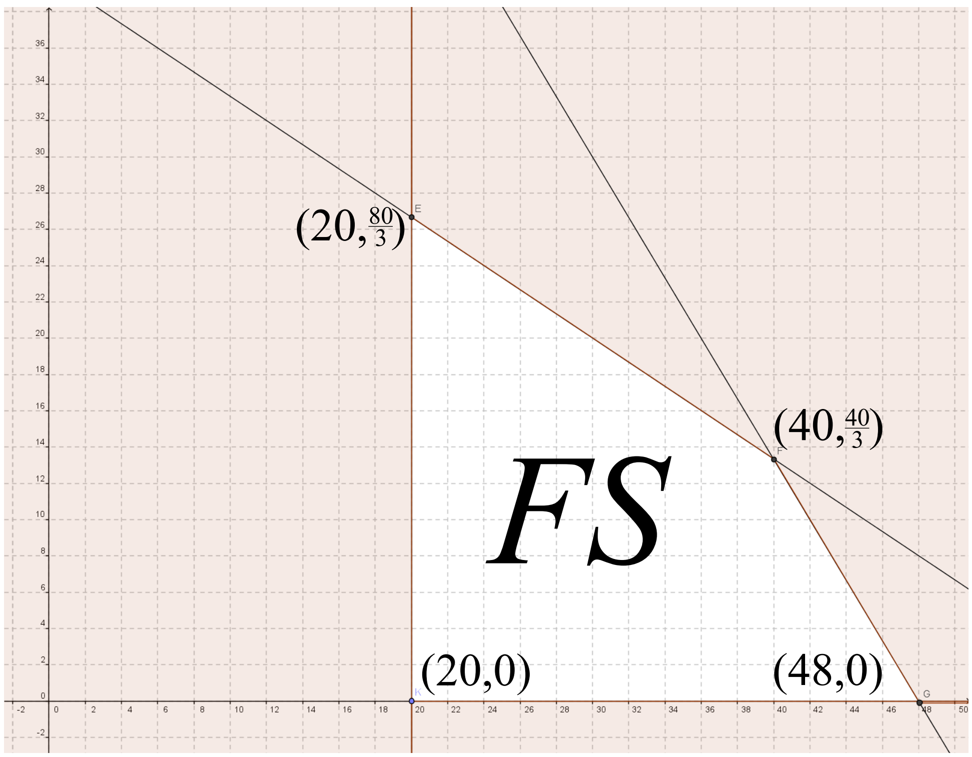
Therefore, z is maximized at every point on the line segment between $(2, 4)$ and $(10, 0)$.

[10] 4. Maximize $P = 0.6x + 0.9y$ subject to

$$8x + 12y \leq 480, \qquad 10x + 6y \leq 480,$$

$$x \geq 20, \qquad x \geq 0, \qquad y \geq 0.$$

Solution:



Check the corner points in the objective function:

x	y	$P = 0.6x + 0.9y$
20	$\frac{80}{3}$	36
40	$\frac{40}{3}$	36
20	0	12
48	0	28.8

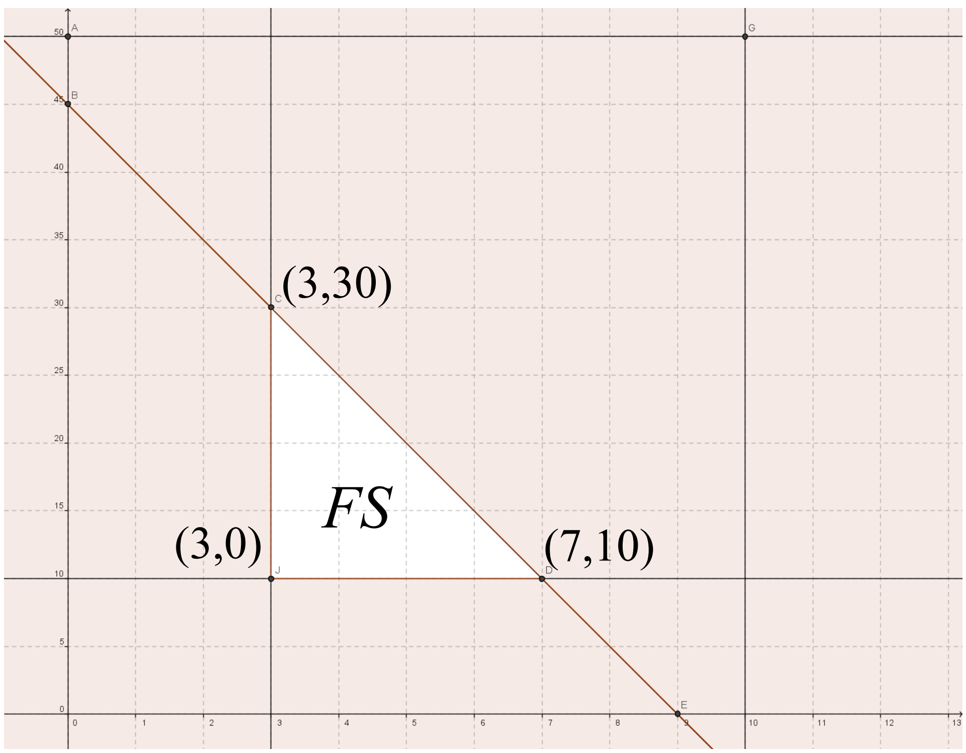
Therefore, P is maximized at every point on the line segment connecting $(20, \frac{80}{3})$ and $(40, \frac{40}{3})$.

[10] 5. A student is taking an exam consisting of 10 essay questions and 50 short-answer questions. He has 90 minutes to take the exam and knows he cannot possibly answer every question. The essay questions are worth 20 points each and the short-answer questions are worth 5 points each. An essay question takes 10 minutes to answer and a short-answer question takes 2 minutes to answer. The student must do at least 3 essay questions and at least 10 short-answer questions. How many of each type of question should the student do to maximize his (potential) mark?

Solution: Let x denote the number of essay questions to do. Let y denote the number of short-answer questions to do.

Then we seek to maximize $M = 20x + 5y$ subject to

$$10x + 2y \leq 90, 0 \leq x \leq 10, 0 \leq y \leq 50, x \geq 3, y \geq 10.$$



Check the corner points in the objective function:

x	y	$M = 20x + 5y$
3	30	210
7	10	190
3	0	60

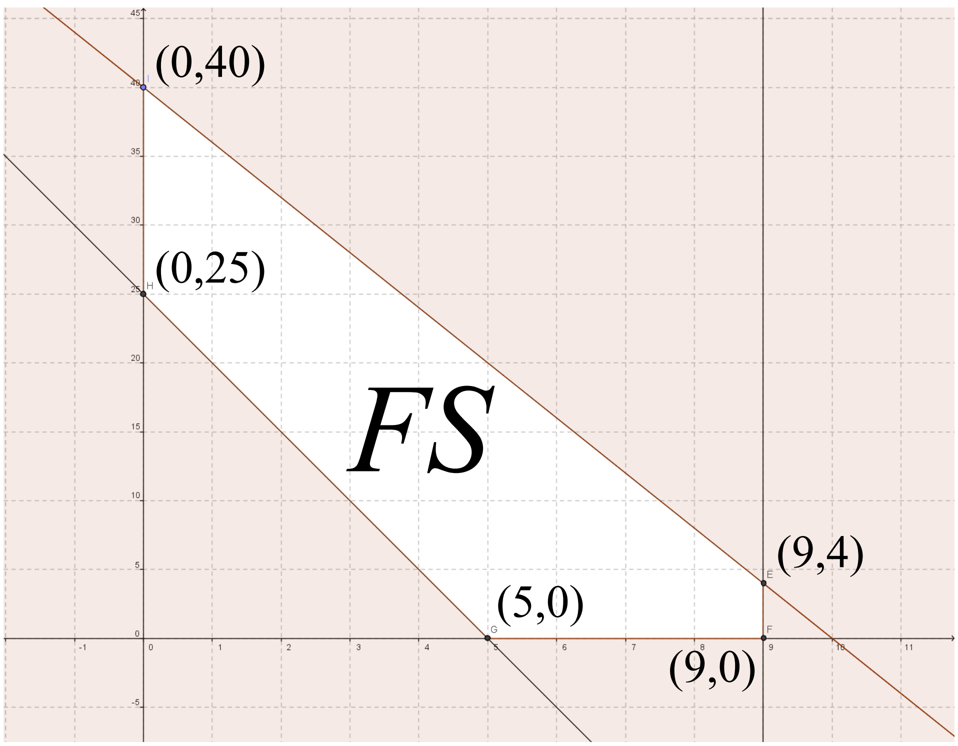
Therefore, M is maximized at $(3, 30)$, indicating that the student should do 3 essay questions and 30 short-answer questions.

[10] 6. A local politician has budgeted at most \$80,000 for her media campaign. She plans to distribute these funds between TV ads and radio ads. Each 1-minute TV ad is expected to be seen by 20,000 viewers and each 1-minute radio ad is expected to be heard by 4,000 listeners. Each minute of TV time costs \$8,000, and each minute of radio time costs \$2,000. She has been advised to use at most 90% of her media campaign budget on television ads. How many minutes of each type of ad should the politician purchase to reach at least 100,000 people at the minimum cost?

Solution: Let x denote the number of TV minutes to purchase. Let y denote the number of radio minutes to purchase.

Then we seek to minimize $C = 8000x + 2000y$ subject to

$$x \geq 0, y \geq 0, 8000x + 2000y \leq 80000, 20000x + 4000y \geq 100000, 8000x \leq 0.9(80000).$$



Check the corner points in the objective function:

x	y	$C = 8000x + 2000y$
0	40	80000
0	25	50000
5	0	40000
9	0	72000
9	4	80000

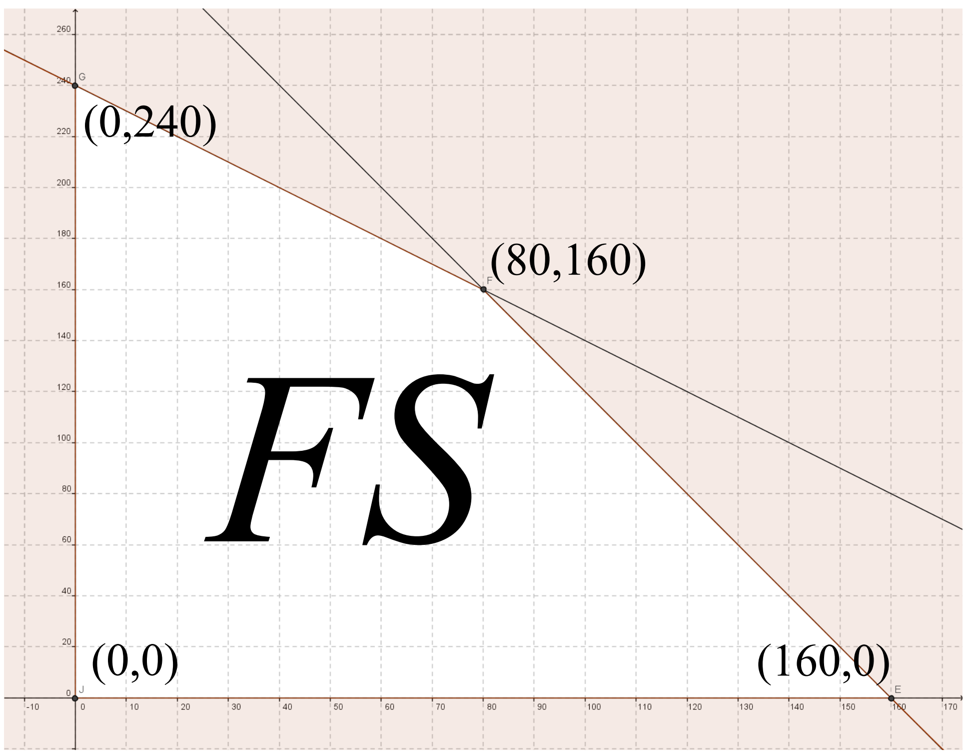
Therefore, C is minimized at $(5, 0)$, indicating that the politician should purchase 5 minutes of TV ads and not bother with radio ads at all.

[10] 7. Suppose you have 240 acres of land. For each acre of corn you plant you will profit \$40, and for each acre of oats you plant, you will profit \$30. However, corn takes 2 hours to harvest, while oats require 1 hour to harvest, and you only have 320 hours available for harvesting. How many acres of each should you plant in order to maximize profits?

Solution: Let x denote the number of acres of corn to plant. Let y denote the number of acres of oats to plant.

Then we seek to maximize $P = 40x + 30y$ subject to

$$x \geq 0, y \geq 0, 2x + y \leq 320, x + y \leq 240.$$



Check the corner points in the objective function:

x	y	$P = 40x + 30y$
0	240	7200
80	160	8000
160	0	6400
0	0	0

Therefore, P is maximized at $(80, 160)$, indicating that you should plant 80 acres of corn and 160 acres of oats.

[10] 8. The Redline company manufactures military trucks in three models: 1 ton, 2 ton and 4 ton at two manufacturing plants referred to here as Plant A and Plant B. The weekly production and production costs are given in the following table:

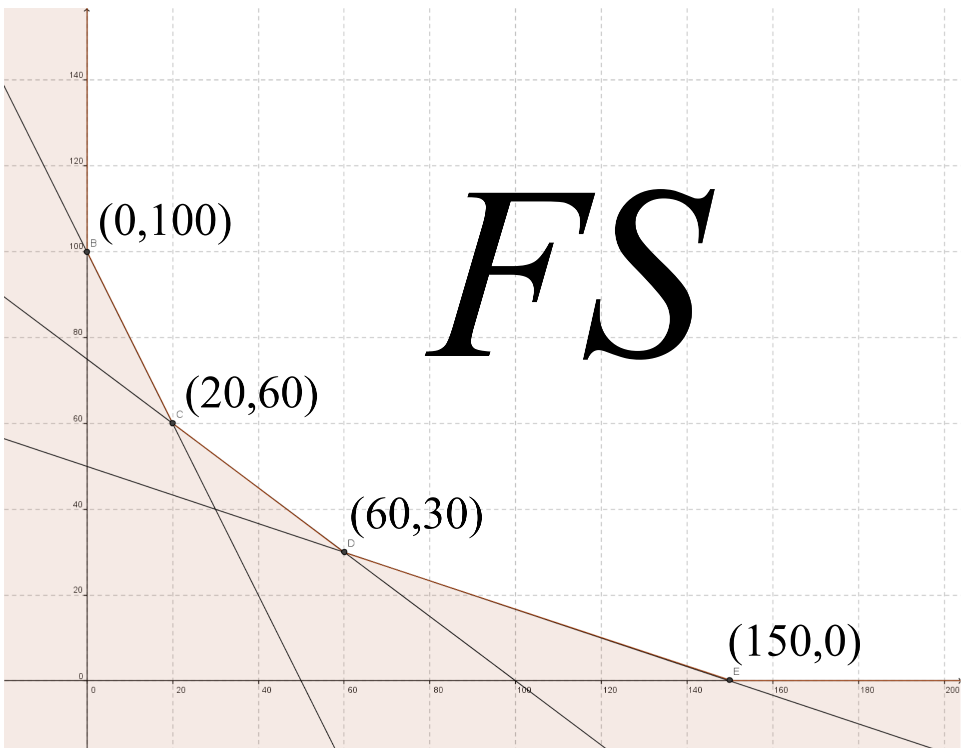
	Plant A	Plant B
1 ton trucks	100	50
2 ton trucks	75	100
4 ton trucks	50	150
Weekly cost	\$4,000,000	\$6,000,000

There is an order from a foreign country to deliver 5000 1 ton trucks, 7500 2 ton trucks, and 7500 4 ton trucks. Find the number of weeks each plant should be operated in order to produce at least the ordered quantities of trucks, at minimum cost.

Solution: Let x denote the number of weeks to run plant A. Let y denote the number of weeks to run plant B.

Then we seek to minimize $C = 4x + 6y$ subject to

$$x \geq 0, y \geq 0, 100x + 50y \geq 5000, 75x + 100y \geq 7500, 50x + 150y \geq 7500.$$



Check the corner points in the objective function:

x	y	$C = 4x + 6y$
0	100	600
20	60	440
60	30	420
150	0	600

Therefore, C is minimized at $(60, 30)$, indicating that Plant A should run for 60 weeks and Plant B should run for 30 weeks.

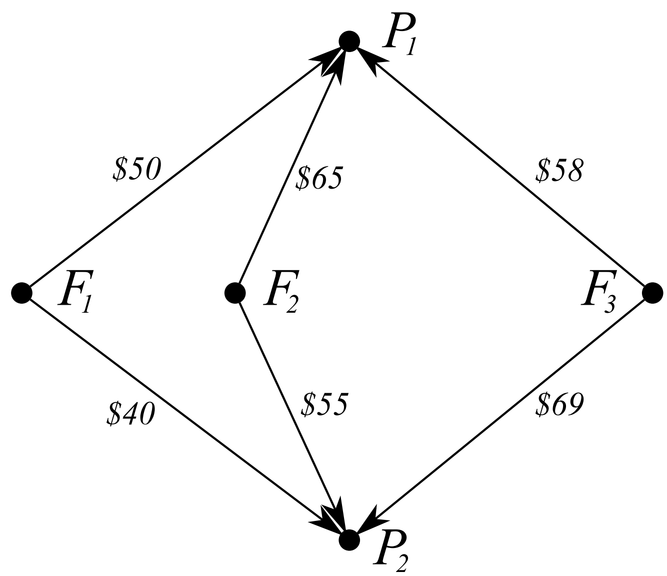
[10] 9. **SET UP BUT DO NOT SOLVE THE FOLLOWING LINEAR PROGRAMMING PROBLEM**. The accompanying schematic diagram shows three farms F_1, F_2, and F_3, each of which grows potatoes and ships them to the two processing plants P_1 and P_2.

Each farm has a maximum production capacity and each plant requires a minimum amount of potatoes for production:

Farm	Max Prod. Capacity
F_1	250 tons
F_2	275 tons
F_3	300 tons.

Plant	Min Prod. Req.
P_1	540 tons
P_2	450 tons.

In addition, each shipping route (from a farm to a processing plant) has a cost associated with shipping each ton of potatoes. The costs in dollars per ton are shown in the diagram below.



Set up but do not solve a linear programming problem associated with minimizing the total shipping cost of the company which owns the two processing plants.

Solution: Let x_1 denote the number of tons of potatoes to ship from F_1 to P_1 Let x_2 denote the number of tons of potatoes to ship from F_2 to P_1 Let x_3 denote the number of tons of potatoes to ship from F_3 to P_1 Let x_4 denote the number of tons of potatoes to ship from F_1 to P_2 Let x_5 denote the number of tons of potatoes to ship from F_2 to P_2 Let x_6 denote the number of tons of potatoes to ship from F_3 to P_2

Then we seek to minimize $C = 50x_1 + 65x_2 + 58x_3 + 40x_4 + 55x_5 + 69x_6$ subject to

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0,$$

$$x_4 \geq 0, x_5 \geq 0, x_6 \geq 0,$$

$$x_1 + x_2 + x_3 \geq 540, x_4 + x_5 + x_6 \geq 450,$$

$$x_1 + x_4 \leq 250, x_2 + x_5 \leq 275, x_3 + x_6 \leq 300.$$