MATH 1300-Borgersen

Quiz 1 Solutions

14R-Term 2 2014-01-28

Name: ______________________________ Student Number: __________________

Answer all questions and show all your work. No calculators allowed. (Total Marks: 21).
You have 20 minutes to complete the quiz.

[4] 1. Find all values for a and b such that $\begin{bmatrix} -1 & 2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} -a & a \\ b & -b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.

Solution:

$$\begin{bmatrix} a+2b & -a-2b \\ 2a+4b & -2a-4b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$a+2b=0, \quad -a-2b=0, \quad 2a+4b=0, \quad -2a-4b=0.$$

$$\left[\begin{array}{cc|c} 1 & 2 & 0 \\ -1 & -2 & 0 \\ 2 & 4 & 0 \\ -2 & -4 & 0 \end{array}\right] \quad \begin{array}{c} R_2 \leftarrow R_2 + R_1 \\ R_3 \leftarrow R_3 - 2R_1 \\ R_4 \leftarrow R_4 + 2R_1 \end{array} \quad \left[\begin{array}{cc|c} 1 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$a = -2t, \quad b = t, \quad t \in \mathbb{R}$$

[3] 2. A linear system of equations has 4 variables, a, b, c, and d. The RREF of the augmented matrix for this system is

$$\left[\begin{array}{cccc|c} 1 & -2 & 0 & 1 & -1 \\ 0 & 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

Find three different solutions for this system:

Solution: There are infinitely many valid solutions. Any three solutions of the form

$$a = -1 - t + 2s, \quad b = s, \quad c = 2 - 3t, \quad d = t, \quad s, t \in \mathbb{R}$$

would suffice. For instance,

Solution #1 : $a = -1, \quad b = 0, \quad c = 2, \quad d = 0 \quad (s = t = 0)$

Solution #2 : $a = 1, \quad b = 1, \quad c = 2, \quad d = 0 \quad (s = 1, t = 0)$

Solution #3 : $a = -2, \quad b = 0, \quad c = -1, \quad d = 1 \quad (s = 0, t = 1)$

[4] 3. Consider the following augmented matrix for a linear system of equations:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & a^2-4 & a-2 \end{array}\right]$$

Find **all values** for a that will result in this system having **infinitely many solutions**. Justify your answer.

Solution: To have infinitely many solutions, the last row must be a zero row, forcing $a^2-4=a-2=0$. The only value for a that makes this true is $a=2$.

4. Simplify each of the following matrix expressions, or state why it can't be done.

[4] (a) $\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^T \begin{bmatrix} -1 & -2 & 0 \\ 0 & 1 & -1 \end{bmatrix}\right)^T$

Solution:

$$\left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^T \begin{bmatrix} -1 & -2 & 0 \\ 0 & 1 & -1 \end{bmatrix}\right)^T = \begin{bmatrix} -1 & -2 & 0 \\ 0 & 1 & -1 \end{bmatrix}^T \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$
$$= \begin{bmatrix} -1 & 0 \\ -2 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ 1 & 0 \\ -3 & -4 \end{bmatrix}.$$

[2] (b) $\left((I_2)^2 + (I_3)^2\right)^T$

Solution:

$$\left((I_2)^2 + (I_3)^2\right)^T = (I_2 + I_3)^T$$

Can't be done, since I_2 is 2×2 and I_3 is 3×3.

[4] 5. Let $A = \begin{bmatrix} -2 \\ 1 \\ -1 \end{bmatrix}$. Find all values for k such that $(kA)^T(kA) = \begin{bmatrix} 1 \end{bmatrix}$.

Solution: $(kA)^T(kA) = \begin{bmatrix} -2k & k & -k \end{bmatrix} \begin{bmatrix} -2k \\ k \\ -k \end{bmatrix} = \begin{bmatrix} 6k^2 \end{bmatrix} = \begin{bmatrix} 1 \end{bmatrix}$. Therefore $6k^2 = 1$ and so $k = \pm\frac{1}{\sqrt{6}} = \pm\frac{\sqrt{6}}{6}$.

6. **BONUS (2 marks)** Find an example of two matrices A and B such that $(AB)^T \neq A^T B^T$. Justify your answer.

Solution: There are infinitely many examples. For instance, if $A_{3,5}$ and $B_{5,2}$, then $(AB)^T$ exists and is 2×3. However, A^T is 5×3, and B^T is 2×5, and therefore $A^T B^T$ doesn't even exist. Equally good would be:

$$\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}^T = \left(\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}\right)^T \neq \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}.$$