MATH 1300-Borgersen

Quiz 4B Solutions

14R-Term 2 2014-03-18

Name: ______________________________ Student Number: __________________

Answer all questions and show all your work. No calculators allowed. (Total Marks: 38). You have 20 minutes to complete the quiz.

1. Let $\mathbf{v} = (1, 2, 4)$, $\mathbf{e} = (3, 2, 1)$, $\mathbf{u} = (3, 2, 4)$. Calculate each of the following:

[5] (a) $\mathrm{proj}_{\mathbf{e}}\,\mathbf{v}$

Solution:

$$\mathrm{proj}_{\mathbf{e}}\,\mathbf{v} = \frac{\mathbf{e} \bullet \mathbf{v}}{||\mathbf{e}||^2}\mathbf{e} = \frac{(3,2,1) \bullet (1,2,4)}{1^2 + 2^2 + 3^2}\mathbf{e}$$

$$= \frac{3+4+4}{9+4+1}(3,2,1) = \frac{11}{14}(3,2,1) = \left(\frac{33}{14}, \frac{11}{7}, \frac{11}{14}\right).$$

[5] (b) $\mathbf{v} \times \mathbf{u}$

Solution:

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 4 \\ 3 & 2 & 4 \end{vmatrix} = \mathbf{i}\begin{vmatrix} 2 & 4 \\ 2 & 4 \end{vmatrix} - \mathbf{j}\begin{vmatrix} 1 & 4 \\ 3 & 4 \end{vmatrix} + \mathbf{k}\begin{vmatrix} 1 & 2 \\ 3 & 2 \end{vmatrix} = (0, 8, -4).$$

[5] (c) The area of the parallelogram determined by $\mathbf{u}$ and $\mathbf{v}$

Solution:

$$\text{Area} = \frac{1}{2}||\mathbf{u} \times \mathbf{v}|| = \frac{1}{2}||\mathbf{v} \times \mathbf{u}|| = \frac{1}{2}||(0,8,-4)|| = \frac{8^2 + 4^2}{2} = \frac{64+16}{2} = \frac{80}{2} = 40.$$

[5] 2. Find all $k \in \mathbb{R}$ such that $(-1, -2, k, -4)$ is orthogonal to $(-2, 5, k, 2)$.

Solution:

$$(-1,-2,k,-4) \bullet (-2,5,k,2) = 2 - 10 + k^2 - 8 = k^2 - 16.$$

To be orthogonal, we need the dot product to equal zero, and so we have $k^2 - 16 = 0$ and so $k = \pm 4$.

3. Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ be vectors in $\mathbb{R}^3$. Which of the following expressions make sense? If they don't, explain why.

[2] (a) $\mathbf{u} \bullet (\mathbf{v} + \mathbf{w})$

Solution: This does make sense.

[2] (b) $||\mathbf{v} \bullet \mathbf{u}||$

Solution: This does NOT make sense. $\mathbf{v} \bullet \mathbf{u}$ is a scalar, which has no length.

[2] (c) $(\mathbf{u} \bullet \mathbf{v}) + \mathbf{u}$

Solution: This does NOT make sense. $\mathbf{u} \bullet \mathbf{v}$ is a scalar, which cannot be added to a vector.

[2] (d) $(\mathbf{u} \bullet \mathbf{v}) \times \mathbf{w}$

Solution: This does NOT make sense. $\mathbf{u} \bullet \mathbf{v}$ is a scalar, which cannot be crossed with a vector.

[2] (e) $(\mathbf{u} \times \mathbf{v}) \times \mathbf{w}$

Solution: This does make sense.

[8] 4. Let $\mathbf{u} = (a, b, c)$, $\mathbf{v} = (d, e, f)$. Prove that $\mathbf{u} \times \mathbf{v}$ is orthogonal to $\mathbf{v}$.

Solution:

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a & b & c \\ d & e & f \end{vmatrix} = \mathbf{i} \begin{vmatrix} b & c \\ e & f \end{vmatrix} - \mathbf{j} \begin{vmatrix} a & c \\ d & f \end{vmatrix} + \mathbf{k} \begin{vmatrix} a & b \\ d & e \end{vmatrix} = (bf - ce, cd - af, ae - bd).$$

$$\begin{aligned} (\mathbf{u} \times \mathbf{v}) \bullet \mathbf{v} &= (bf - ce, cd - af, ae - bd) \bullet (d, e, f) \\ &= d(bf - ce) + e(cd - af) + f(ae - bd) \\ &= bdf - cde + cde - aef + aef - bdf \\ &= 0. \end{aligned}$$