

Name: _____

Student Number: _____

Answer all questions and show all your work. No calculators allowed. (Total Marks: 26).
You have 20 minutes to complete the quiz.

- [8] 1. Find a and b so that $f(x) = x^3 + ax^2 + bx$ has critical numbers $x = 1$ and $x = 3$.

Solution:

$$f'(x) = 3x^2 + 2ax + b.$$

$$f'(1) = 3(1) + 2a(1) + b = 0$$

$$f'(3) = 3(9) + 2a(3) + b = 0$$

$$2a + b = -3$$

$$6a + b = -27$$

Subtract down and we get

$$-4a = 24$$

$$a = -6$$

Plugging back in, we get

$$2(-6) + b = -3$$

$$-12 + b = -3$$

$$b = 9$$

Thus $a = -6$, $b = 9$, and so $f(x) = x^3 - 6x^2 + 9x$.

- [6] 2. Find $f'(x)$ if $f(x) = (e^x + 1)^x$.

Solution:

$$y = (e^x + 1)^x$$

$$\ln y = \ln((e^x + 1)^x)$$

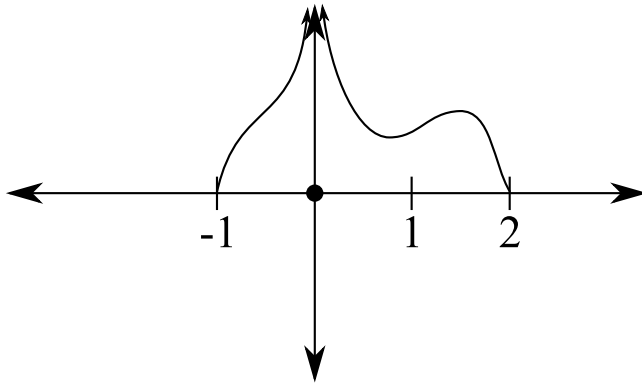
$$\ln y = x \ln(e^x + 1)$$

$$\frac{1}{y} y' = \ln(e^x + 1) + x \frac{1}{e^x + 1} (e^x)$$

$$y' = (e^x + 1)^x \left(\ln(e^x + 1) + \frac{xe^x}{e^x + 1} \right).$$

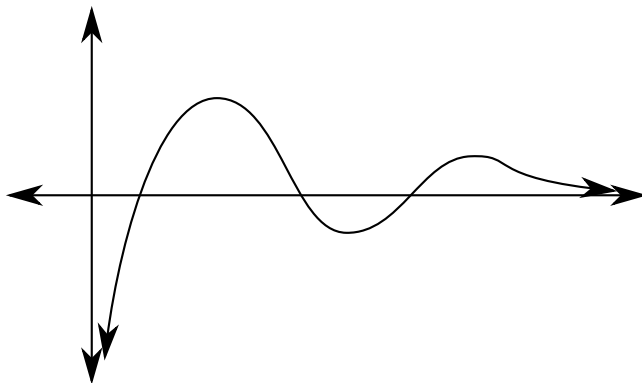
- [4] 3. (a) Sketch a graph of a function with domain $[-1, 2]$ that has exactly one local maximum but no absolute maximum.

Solution: There are lots of possible solutions. For instance:



- [4] (b) Sketch a graph of a function with domain $(0, \infty)$ that has exactly two local maximums, exactly one local minimum, and no absolute minimum.

Solution: There are lots of possible solutions. For instance:



- [4] 4. What are the absolute minimum and maximum values of the function $f(x) = \sqrt{4 - x^2}$? Justify your answer.

Solution: The graph is just the top half of the circle centered at the origin with radius 2. The absolute maximum is the top of the circle at $x = 0$ with value 2, and the absolute minimums are at both end points $x = 2$ and $x = -2$, and have value 0.