Finding the Resultant of A Group of Forces

This is the porch of Antoni Gaudí's Güell Chapel.

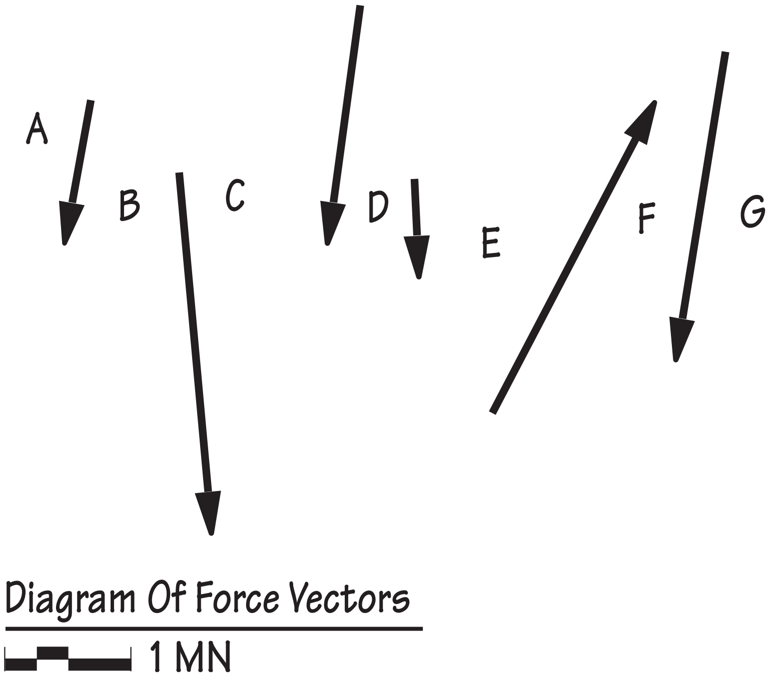
The sloping masonry piers lie along the lines of action of the resultants of the forces from the brick arches and vaults that converge from above. Thus they experience only axial forces under the loading for which they are designed.

In this lesson, you will learn the graphical method for finding the resultant of a group of forces, varying in magnitude and direction, that lie in a single plane.

*To proceed with this lesson, click on the **Next** button here or at the top of any page.*

*When you are done with this lesson, click on the **Contents** button here or at the top of any page to return to the list of lessons.*

Finding the Resultant of A Group of Forces



The Problem: The forces represented by the vectors on this page all vary in magnitude and direction.

We would like to find their resultant, including:

- the magnitude of the resultant force,
- its direction, and
- the location of its line of action.

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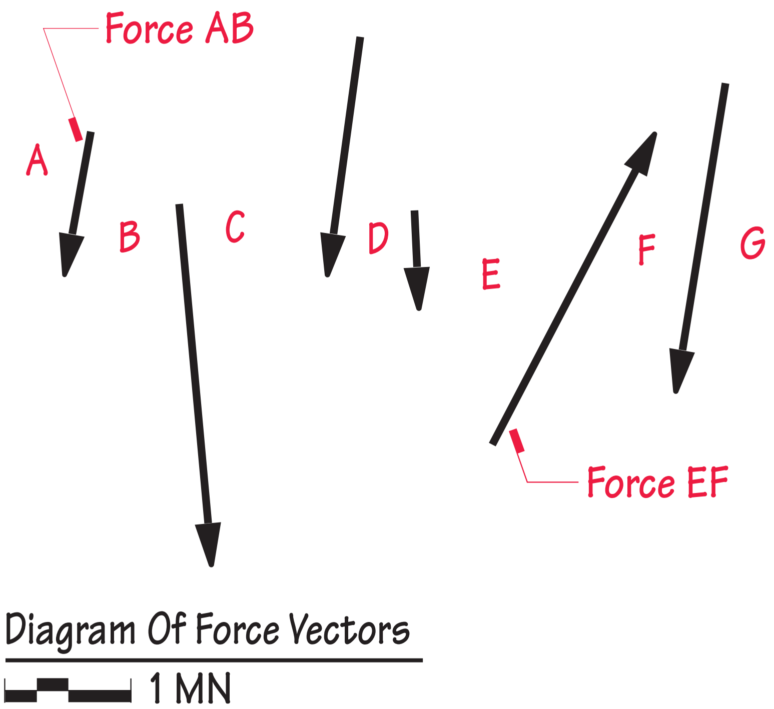

Step 1: Apply interval notation to the Diagram of Force Vectors.

Working left to right, we place capital letters in the intervals between the forces.

Each force is named by the letters on either side. For instance, the leftmost force is *AB*. The force acting upward to the right is *EF*.

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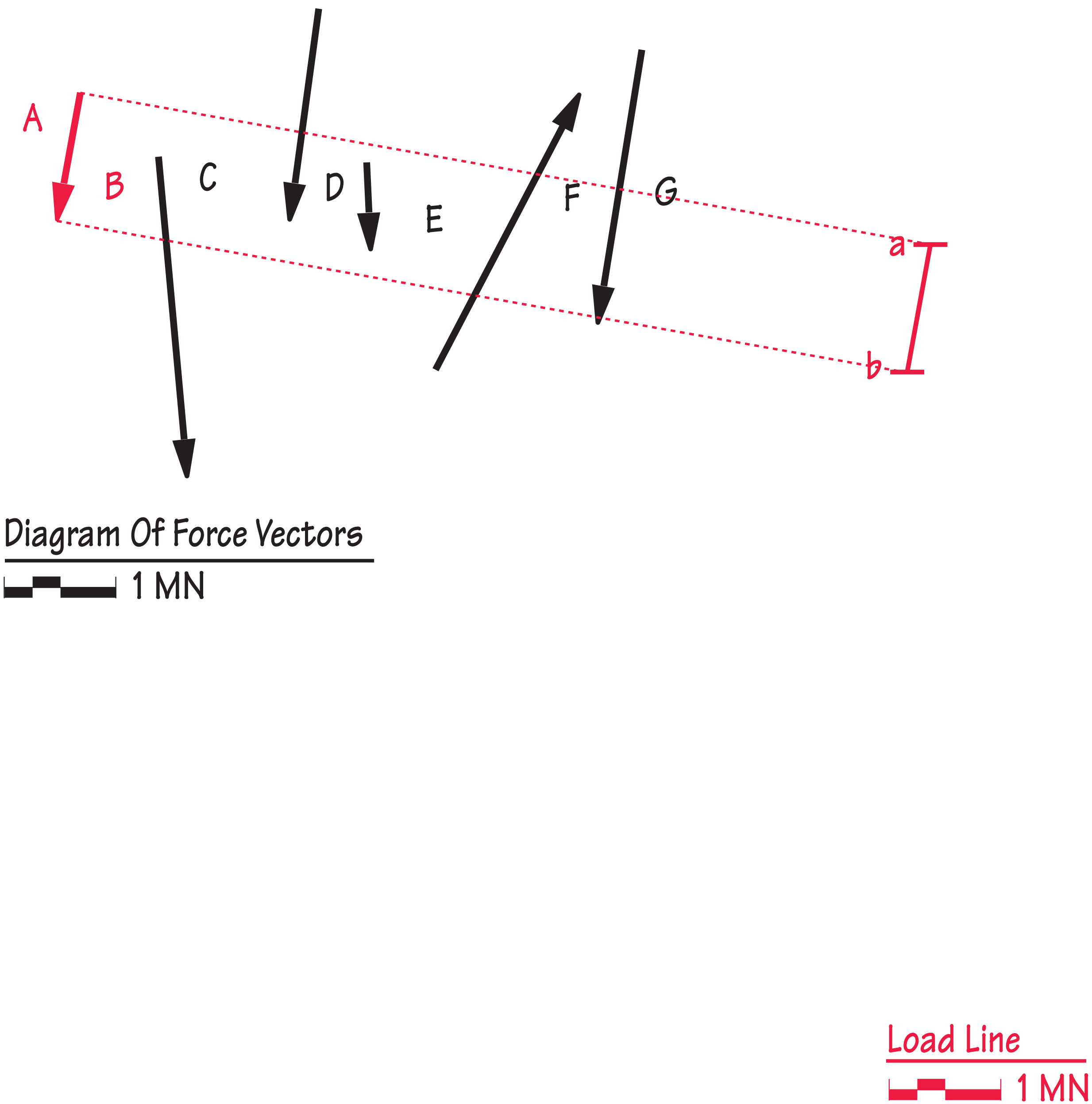


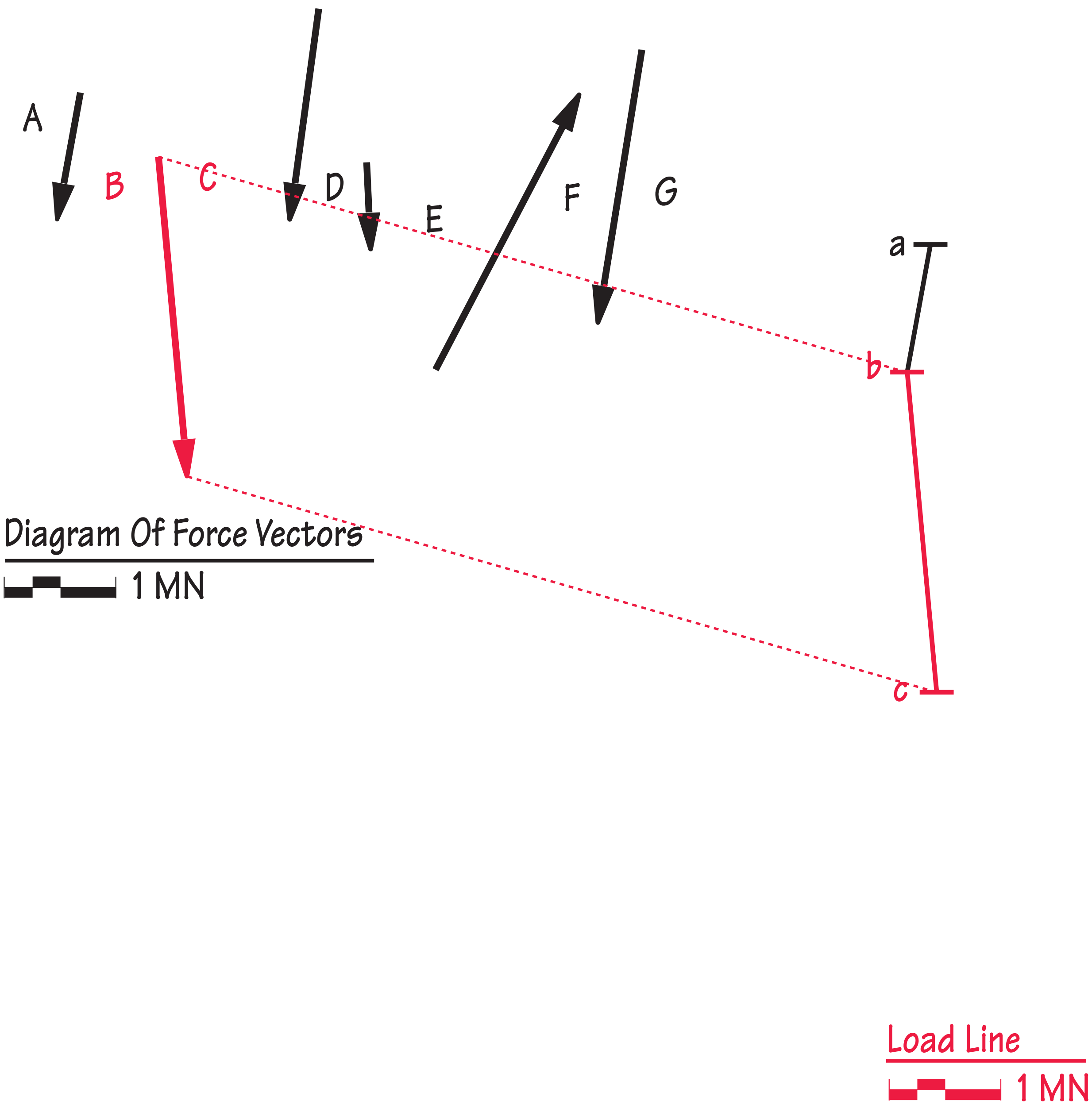
Step 2: Construct the Load Line.

Now we combine the vectors into a load line, using interval notation to keep track of them.

Working again from left to right, we plot the forces tip-to-tail. The first force, *AB*, is plotted onto the Load Line as segment *ab*. Segment *ab* is parallel, and equal in magnitude, to *AB* on the Diagram of Force Vectors.

Because *AB* acts downward, point *b* on the Load Line is below point *a*.

Finding the Resultant of A Group of Forces

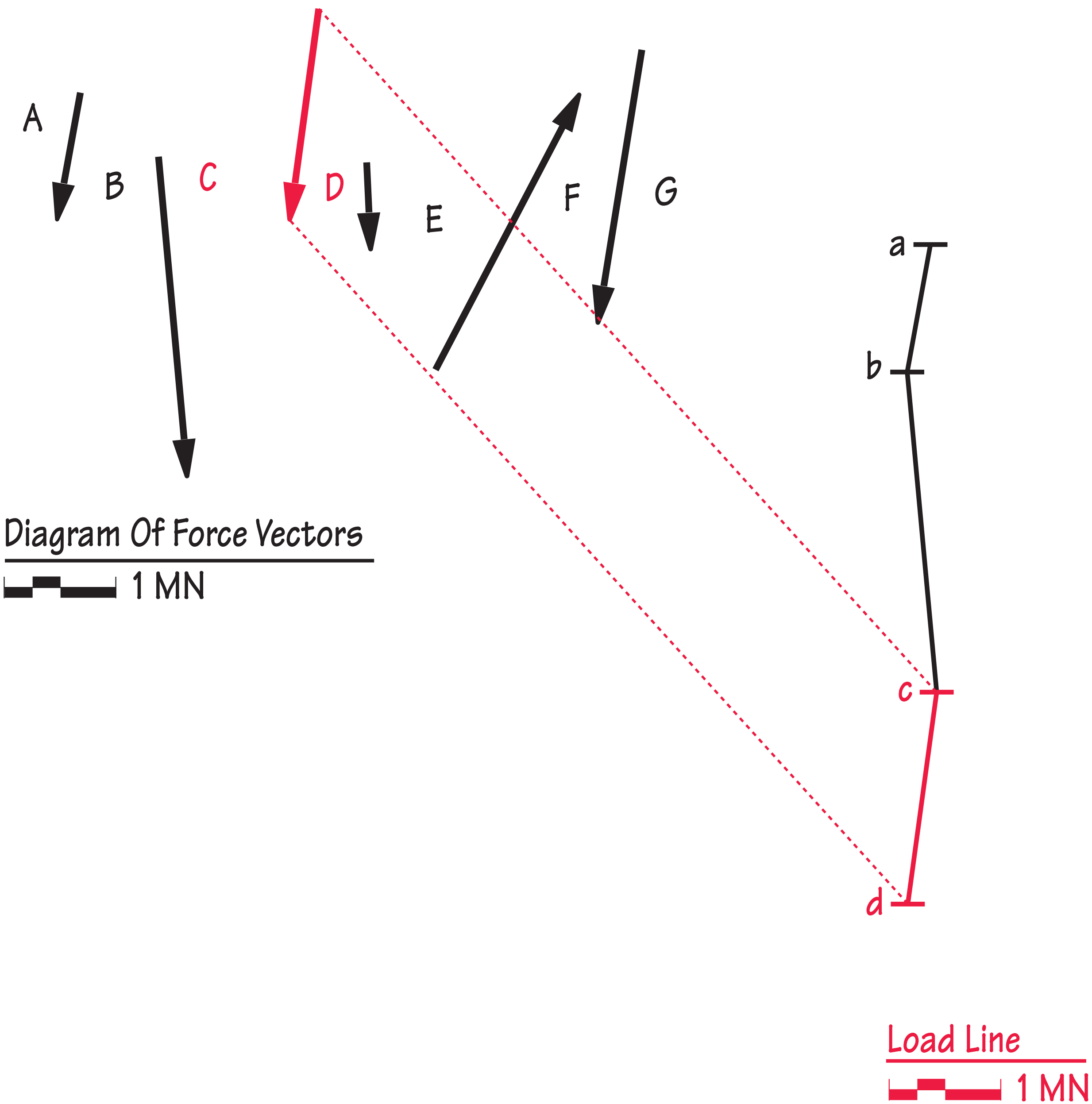


Next we plot *BC* on the Diagram of Forces as segment *bc* on the Load Line.

Again, *bc* is parallel, and equal in magnitude, to *BC*.

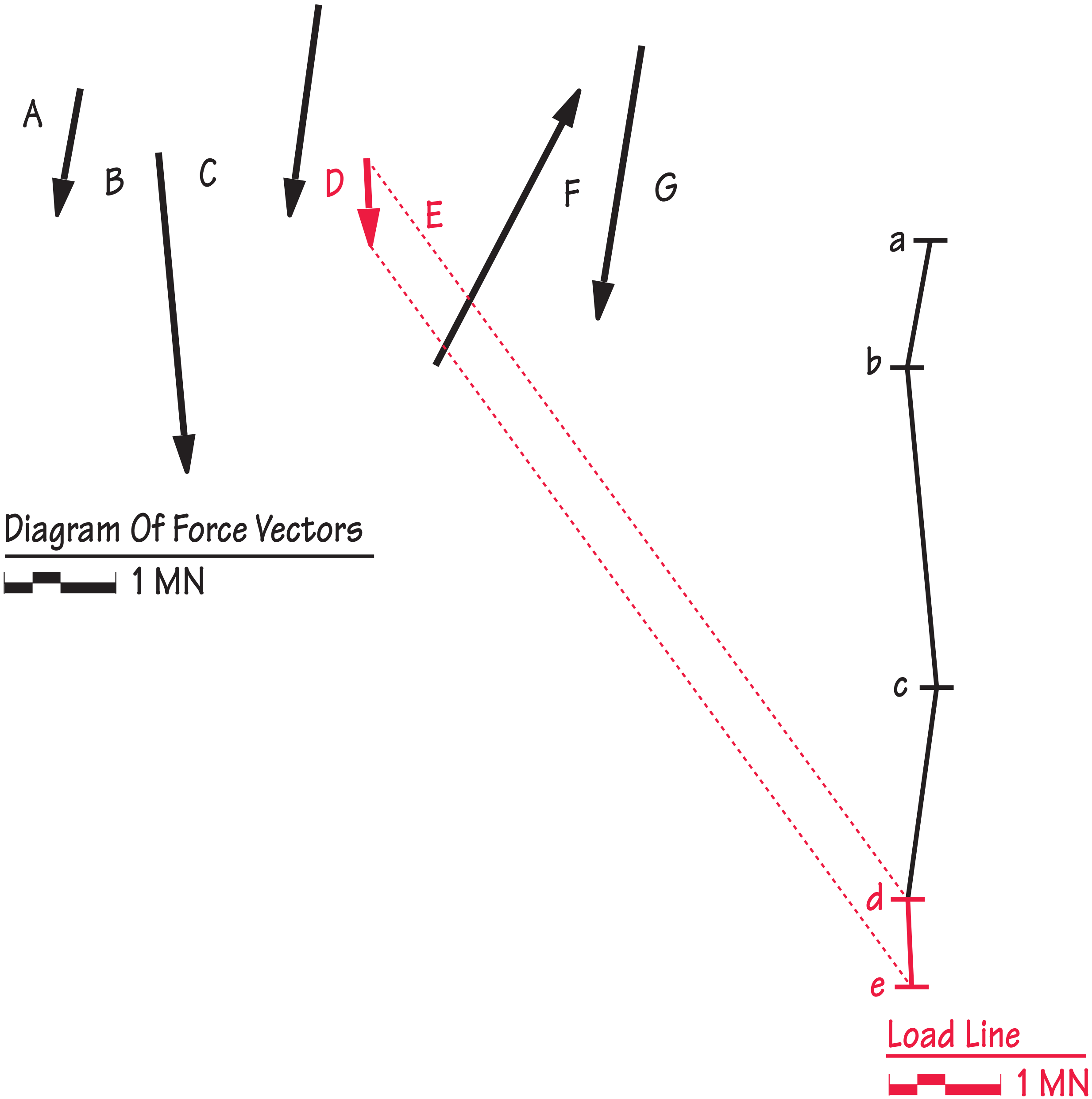
Finding the Resultant of A Group of Forces

Next we plot *cd* onto the Load Line.



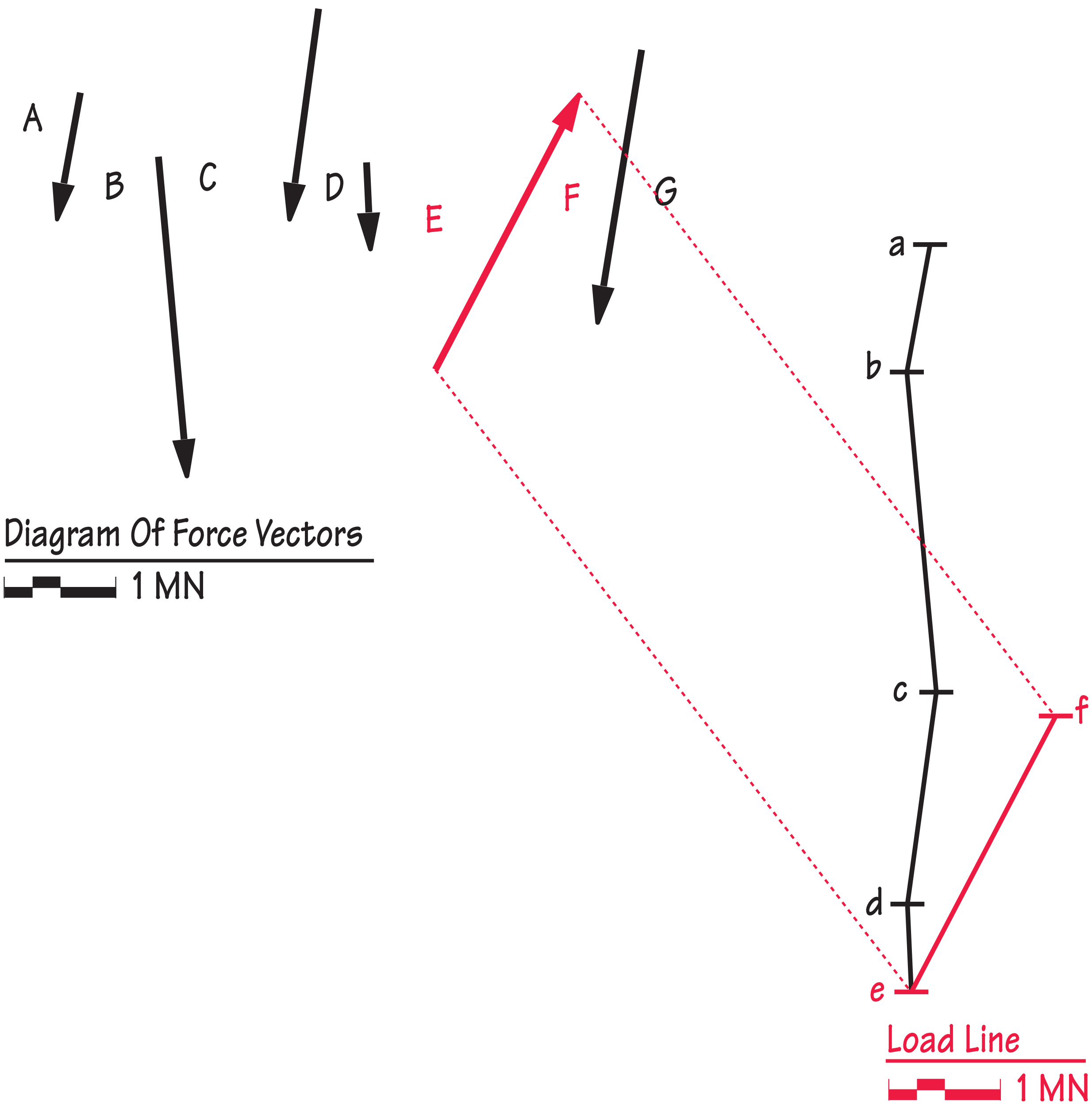
Finding the Resultant of A Group of Forces

Next we plot *de*.

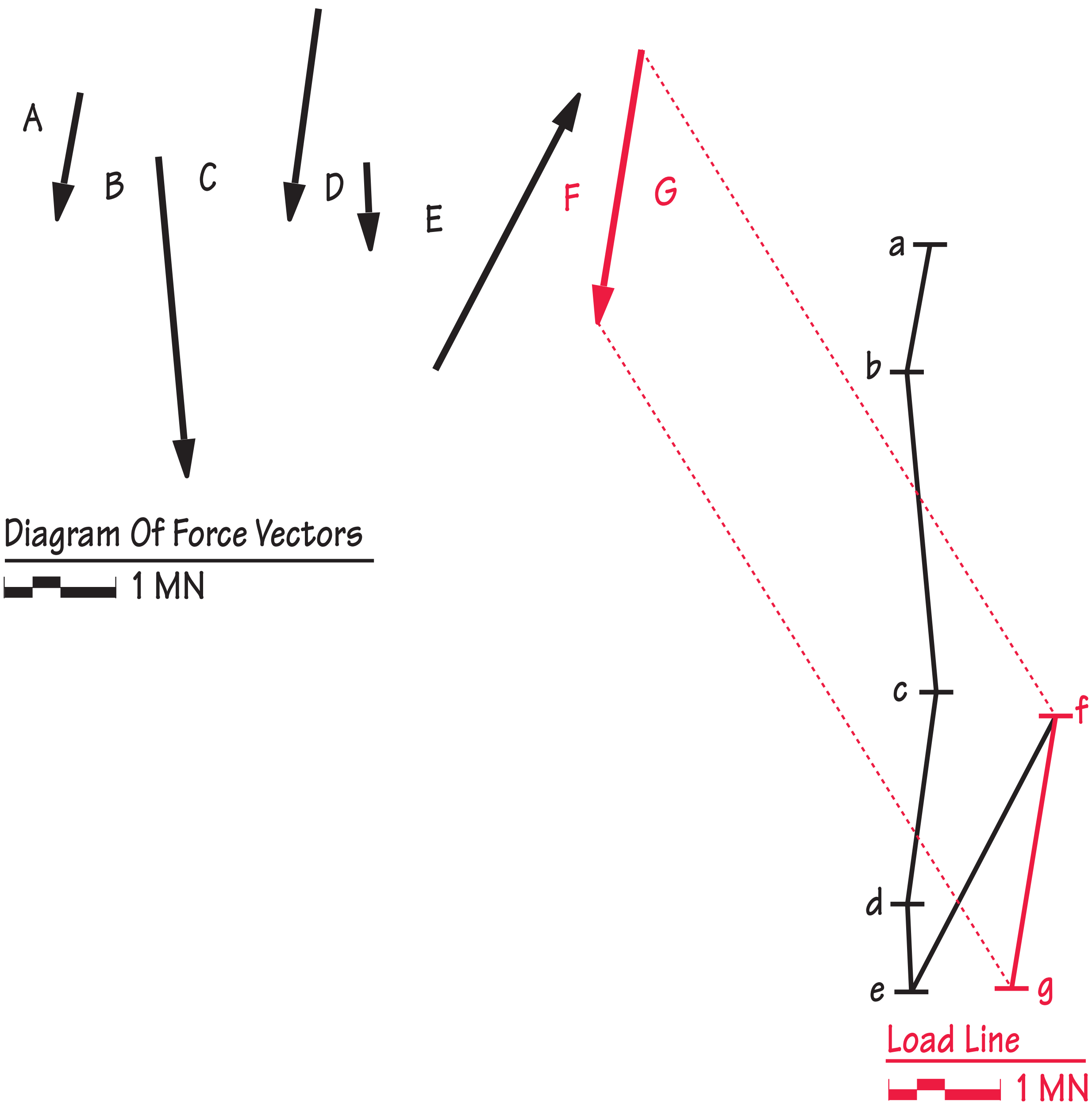


Finding the Resultant of A Group of Forces

Because *EF* acts upward on the Diagram of Force Vectors, point *f* occurs above point *e* on the Load Line.



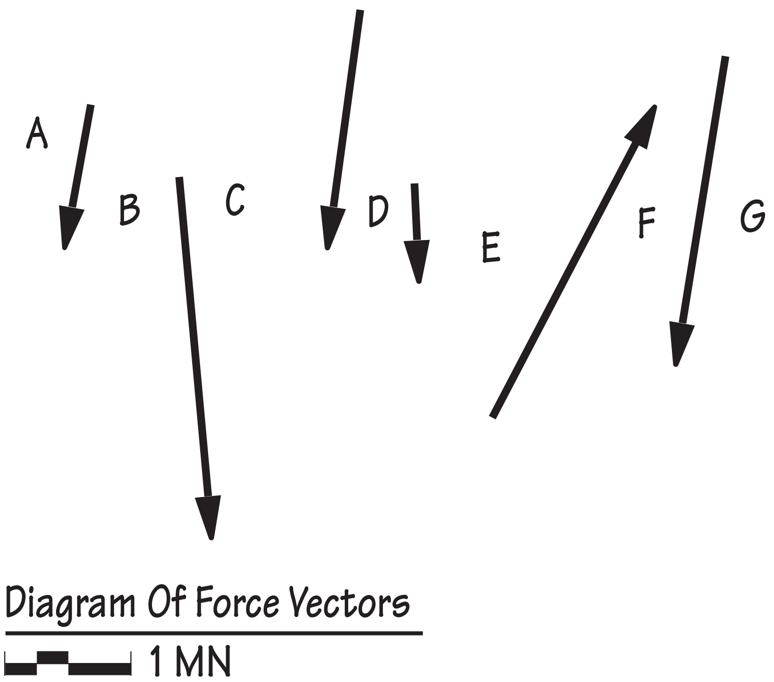
Finding the Resultant of A Group of Forces

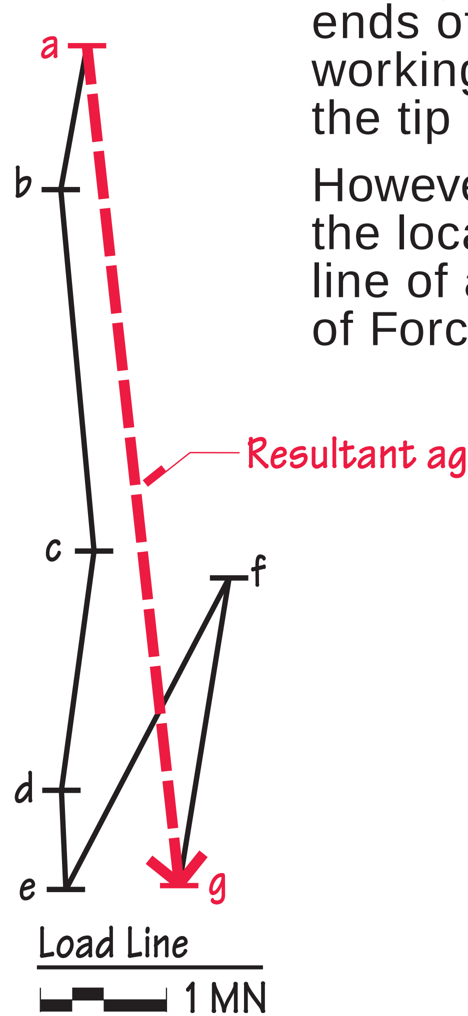


Plotting *fg* completes the Load Line.

Notice that this Load Line is not vertical. This is because its forces have horizontal as well as vertical components.

Finding the Resultant of A Group of Forces



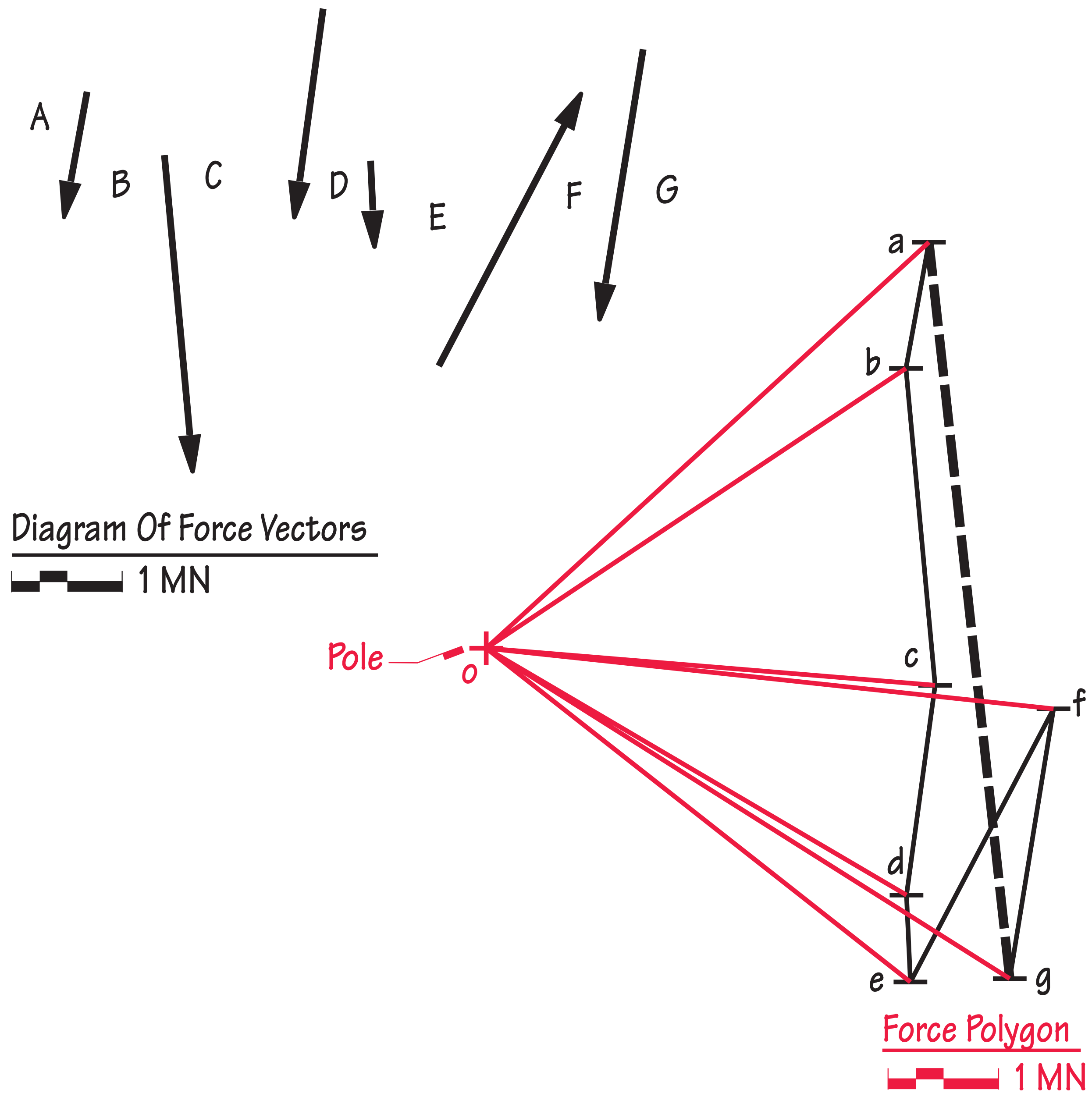


Step 3: Find the magnitude and direction of the resultant.

The magnitude and direction of the resultant, *ag*, is found merely by connecting the ends of the Load Line, working from the tail of *ab* to the tip of *fg*.

However, we do not yet know the location of the resultant's line of action on the Diagram of Force Vectors.

Finding the Resultant of A Group of Forces

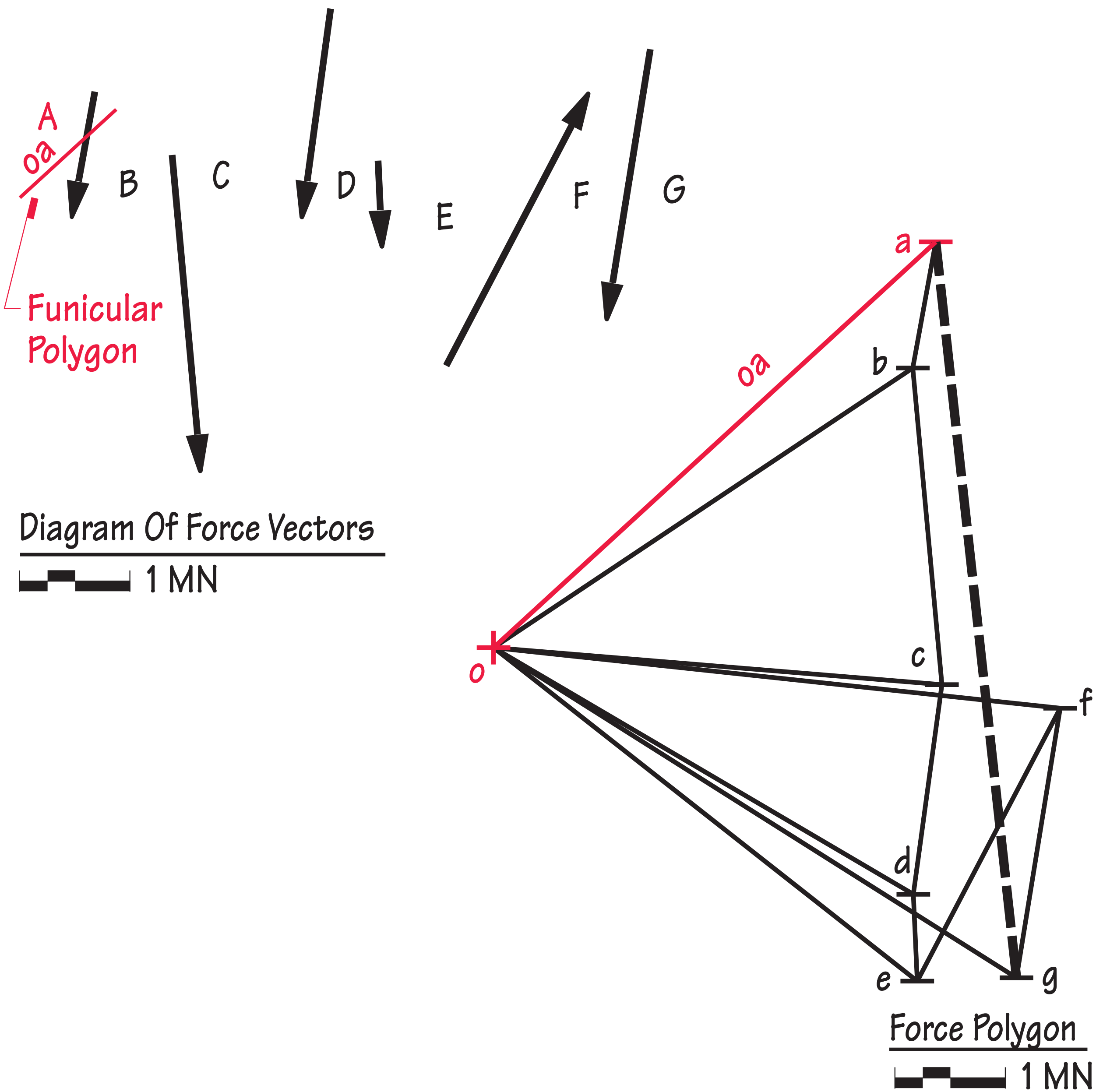


Step 4: Construct a force polygon.

To find the location of the resultant's line of action, we begin by establishing a pole, *o*, at any convenient location near the Load Line.

We draw rays from this pole to the nodes on the Load Line. The Load Line now becomes part of a Force Polygon.

Finding the Resultant of A Group of Forces

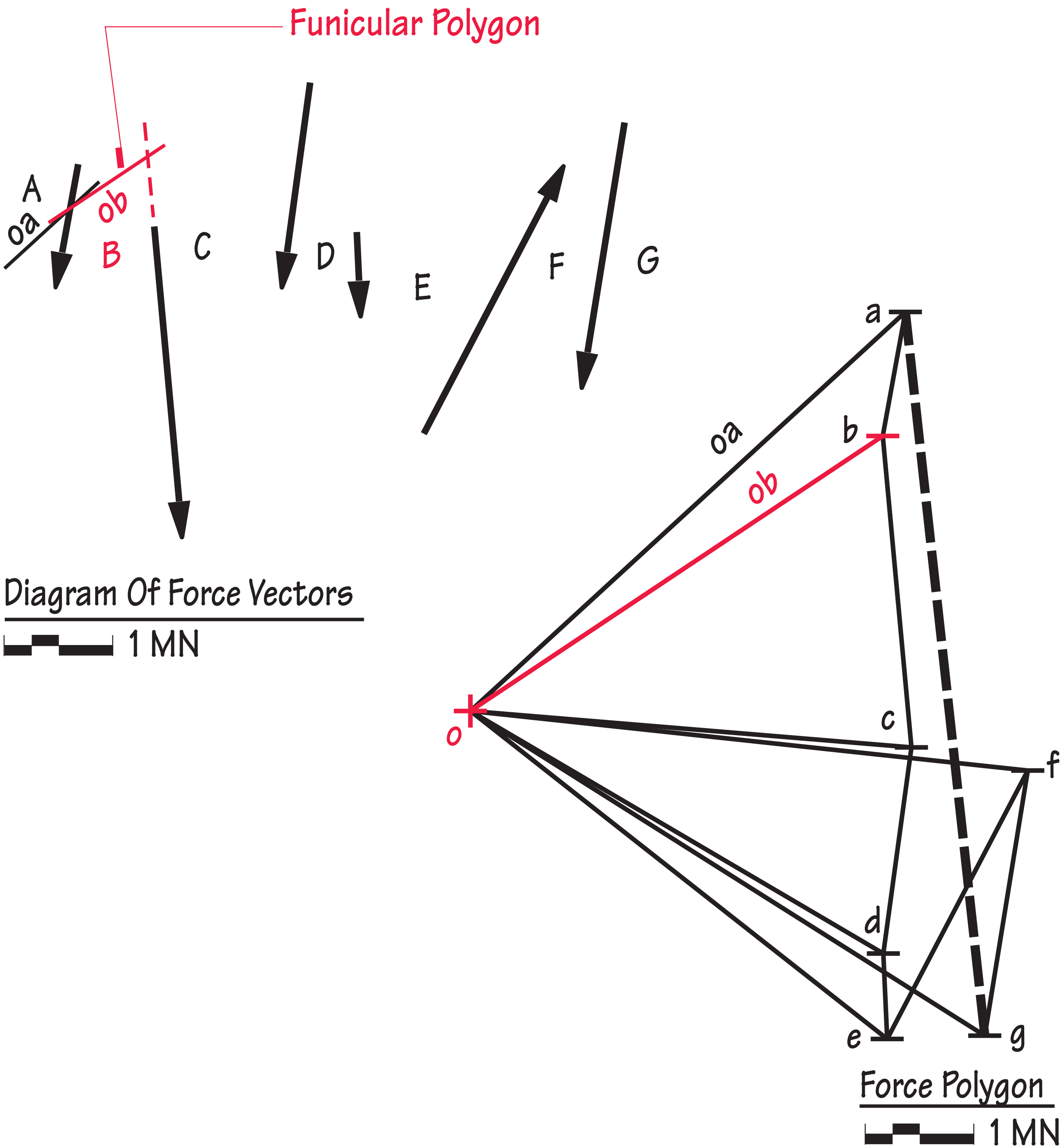


Step 5: Construct the funicular polygon.

Drawing lines parallel to the rays, and using interval notation to maintain order, we construct a Funicular Polygon over the original Diagram of Force Vectors.

Beginning at the left, a line parallel to ray *oa* is drawn in interval *A* of the Diagram of Force Vectors. This line, also labeled *oa,* is placed so that it intersects with force *AB* close to its right-hand end.

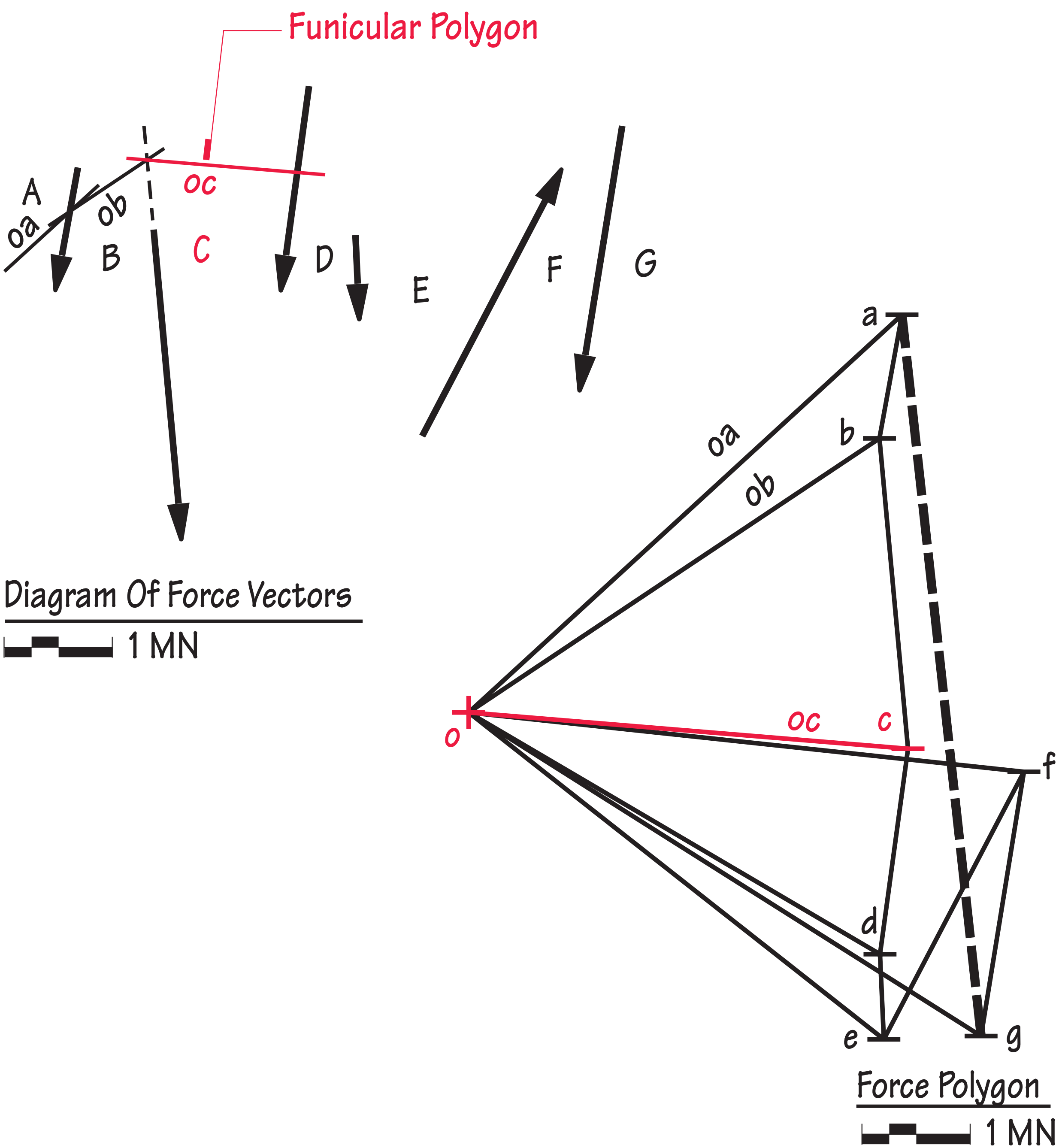
Finding the Resultant of A Group of Forces



Next, a line parallel to ray *ob* is drawn in space *B* of the Diagram of Force Vectors. The left end of this line is drawn to coincide with the intersection of *oa* and *AB*.

The line of action of force *BC* is extended as necessary to intersect with opposite end of *ob*.

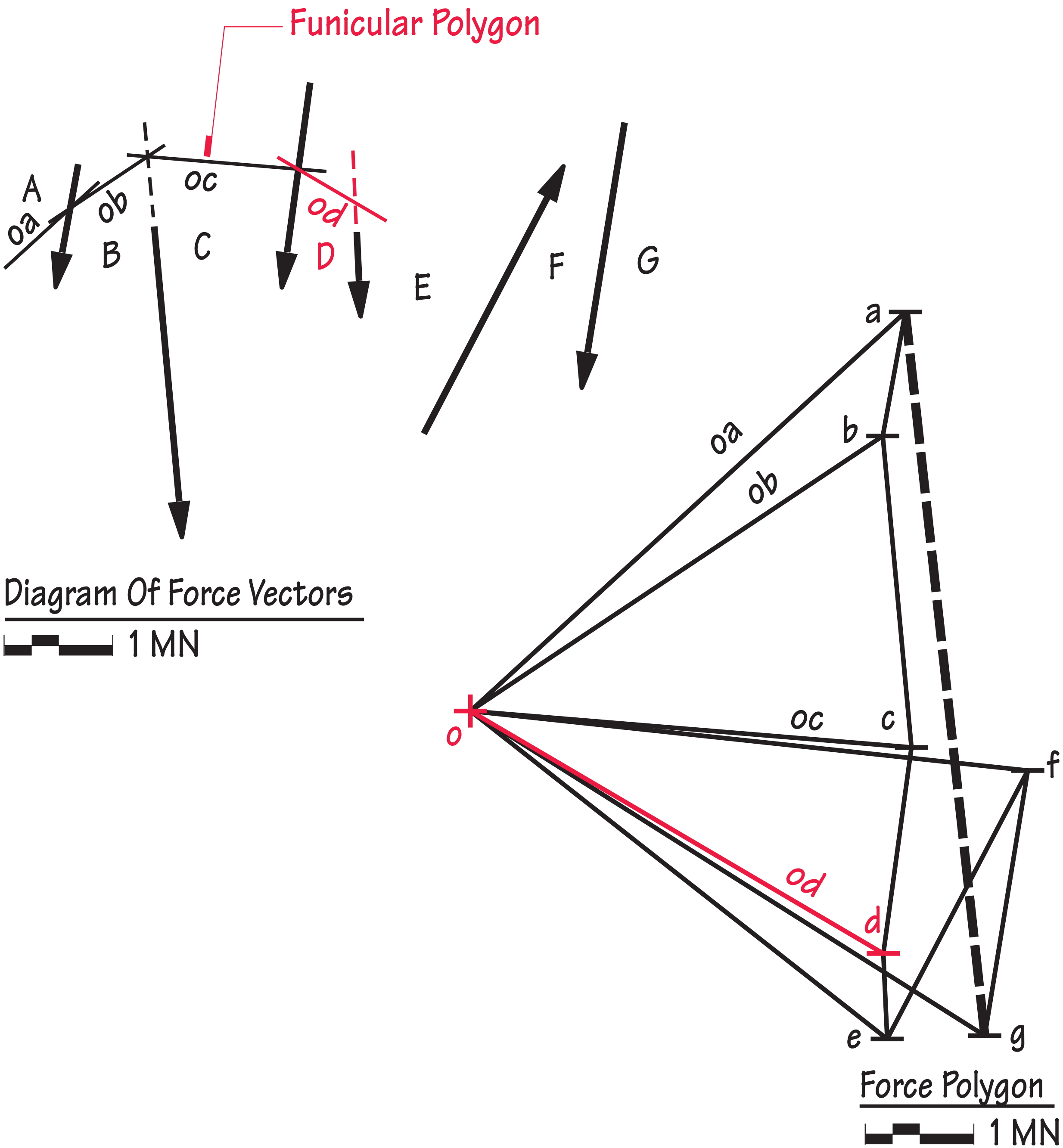
Finding the Resultant of A Group of Forces



Continuing to work from left to right, we next construct *oc* in space *C* of the Diagram of Force Vectors.

The left end of this line coincides with the intersection of *ob* and the line of action of *BC*.

Finding the Resultant of A Group of Forces



Next, we construct *od* in space *D* of the Diagram of Force Vectors.

Once again, we extend the line of action of force *DE* as necessary to intersect with *od*.

Finding the Resultant of A Group of Forces

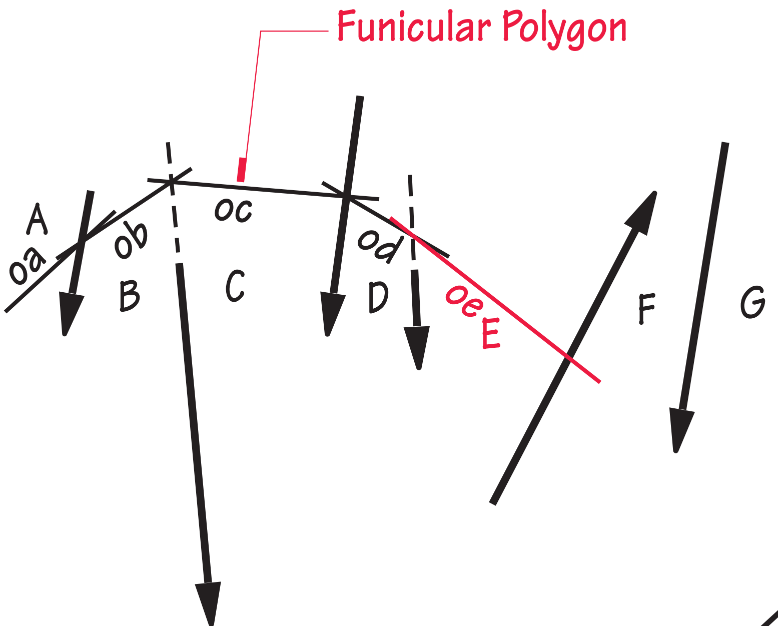
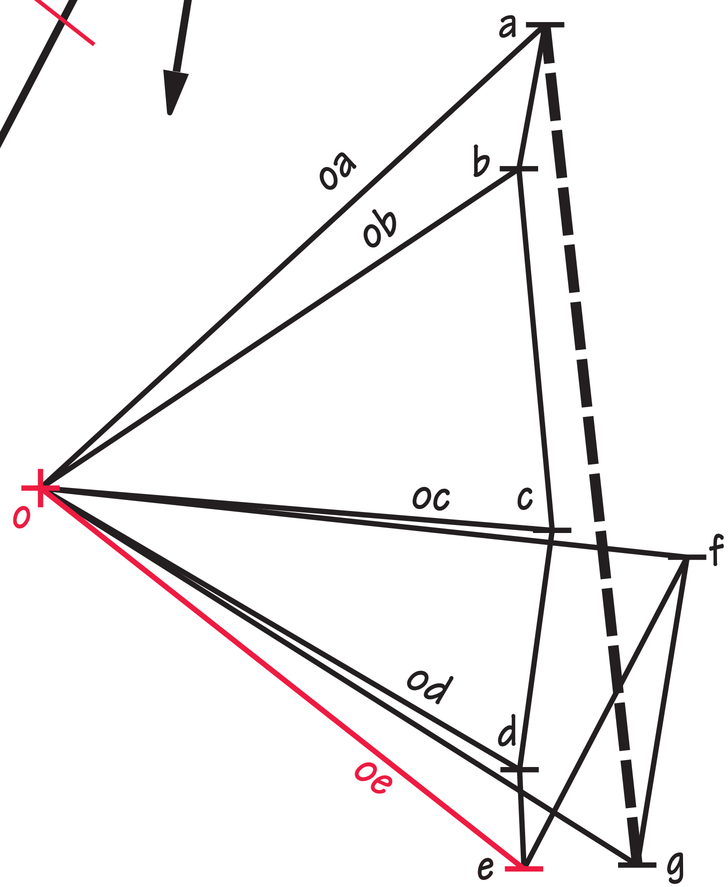


Diagram Of Force Vectors

1 MN

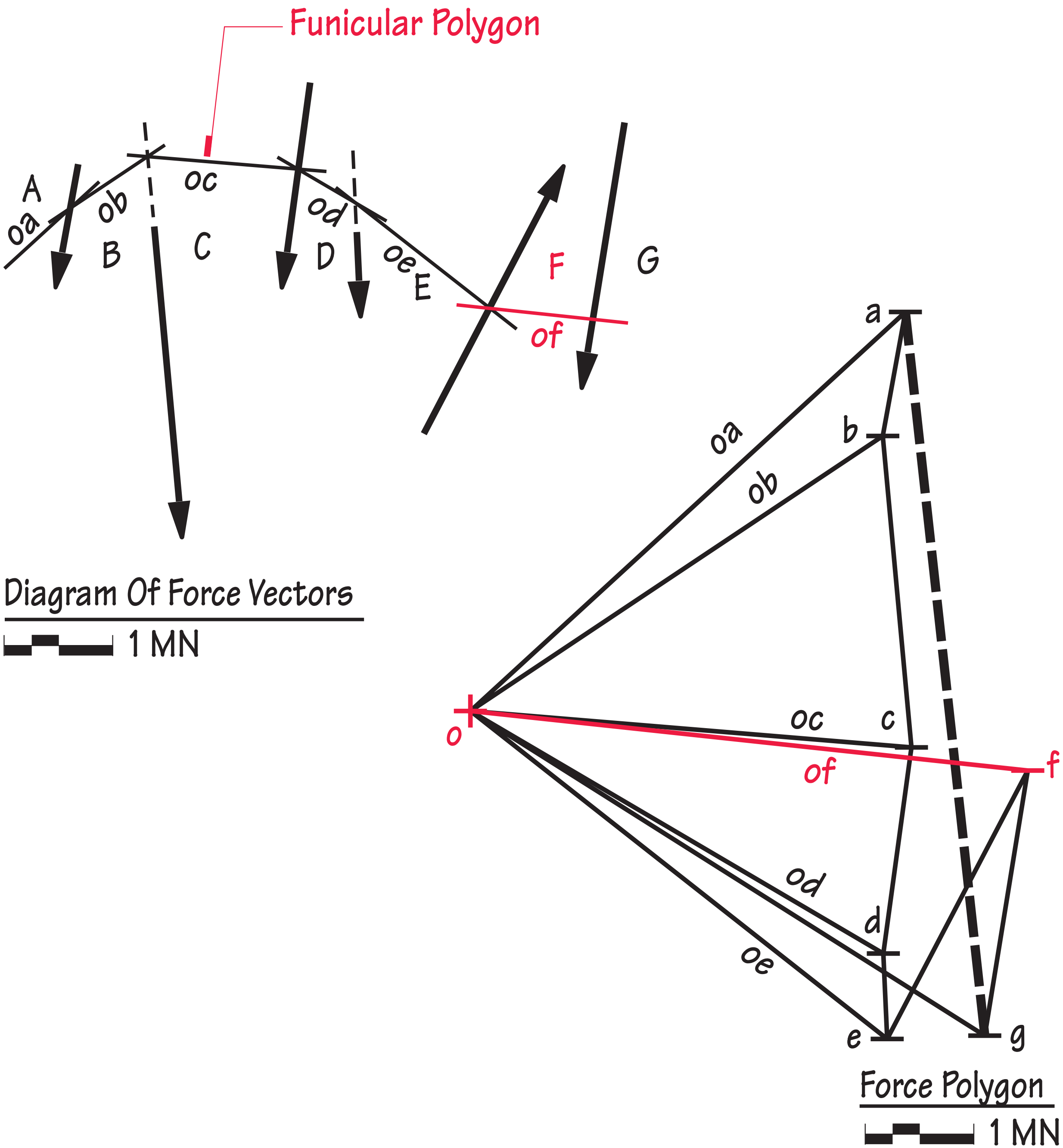


Force Polygon

1 MN

Next, we construct *oe* in space *E* of the Diagram of Force Vectors.

Finding the Resultant of A Group of Forces

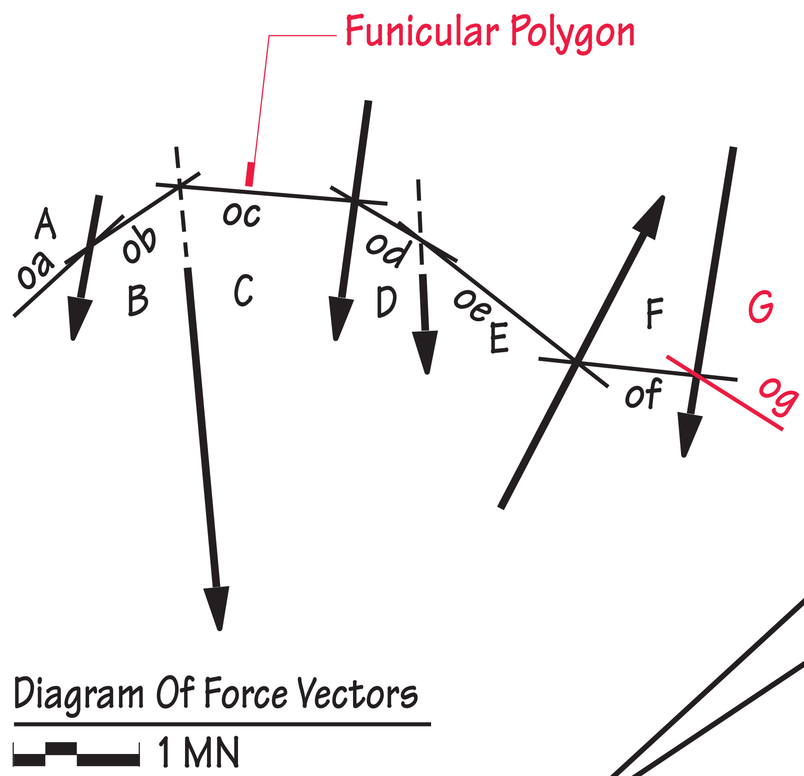


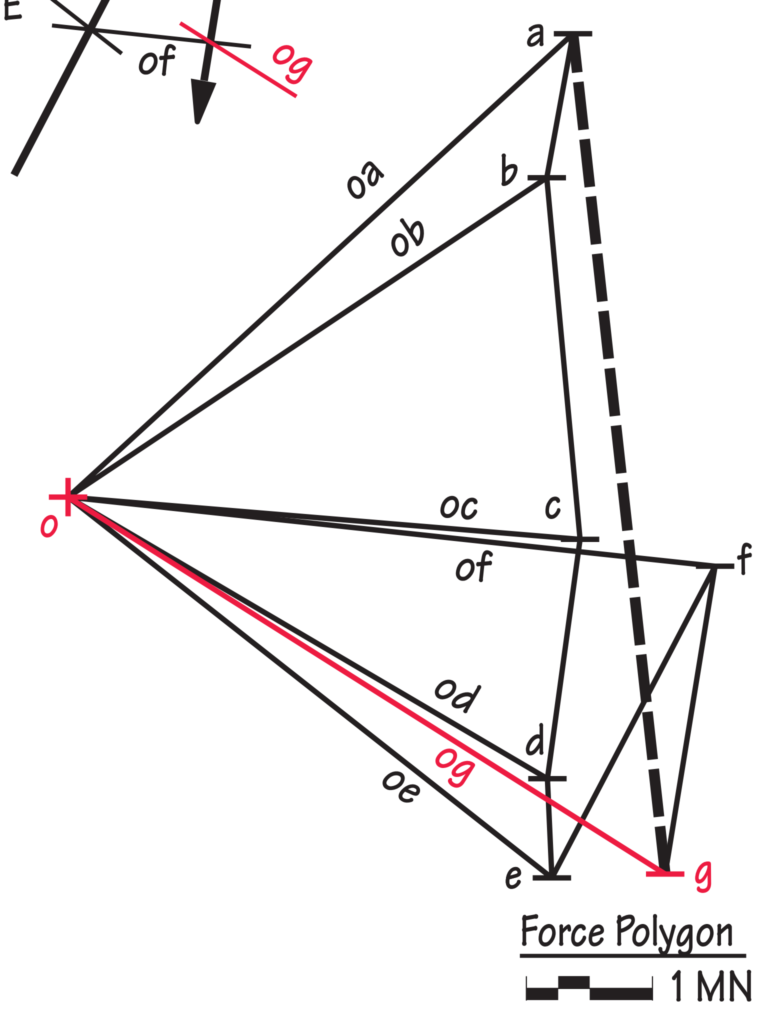
Next, we construct *of*.

Finding the Resultant of A Group of Forces

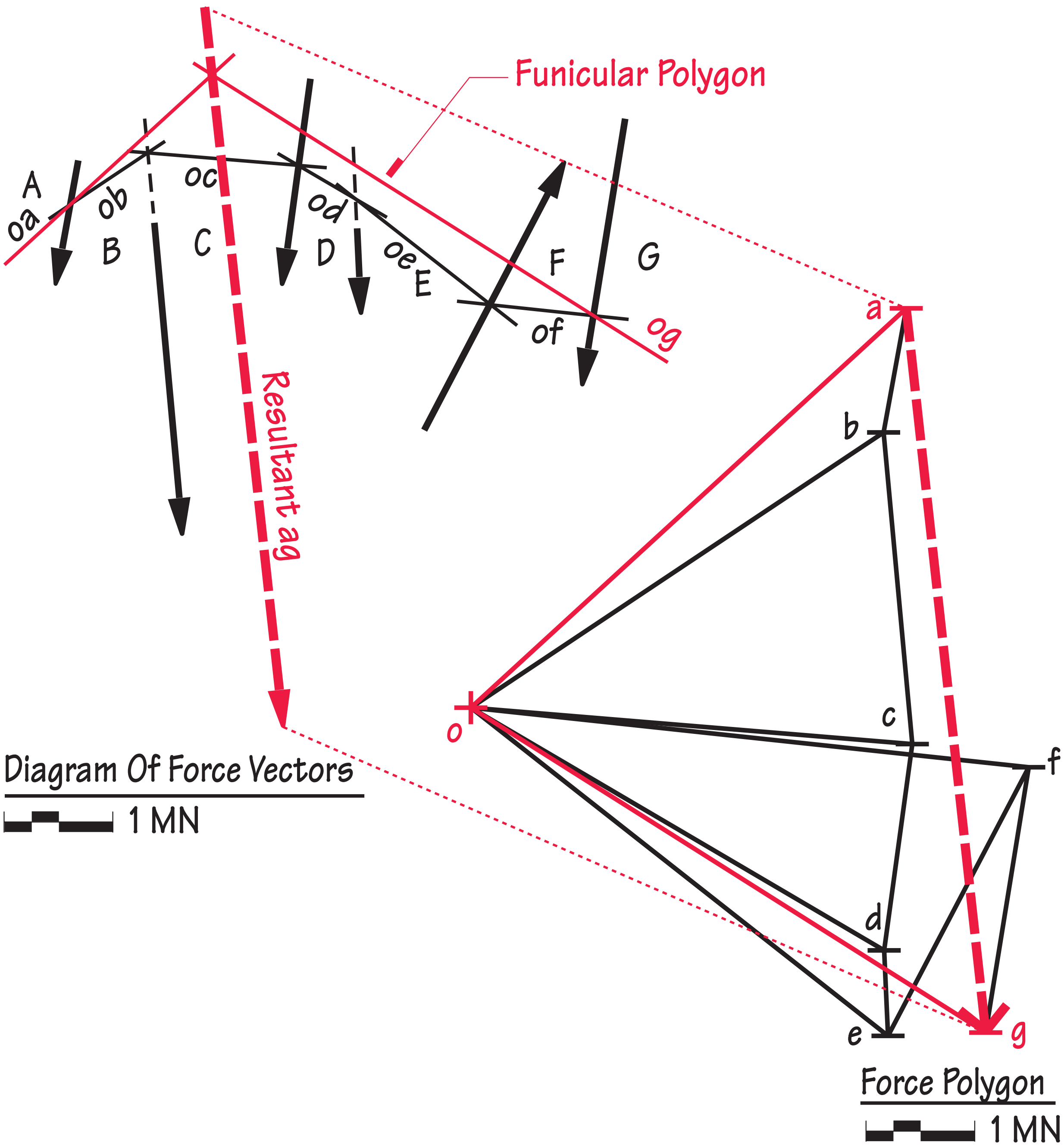
Lastly, we construct *og*.

With all rays of the Funicular Polygon constructed, we can now find the location of the line of action of the resultant.





Finding the Resultant of A Group of Forces



Step 6: Find the line of action of the resultant.

When the first and last segments of the Funicular Polygon, *oa* and *og*, are extended, they intersect on the line of action of force *ag*, the resultant.

The direction and magnitude of *ag* are transferred to the Diagram of Force Vectors from the Force Polygon.

Our solution is complete.

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This technique is especially useful for finding the resultant of parallel or nearly parallel forces. It is a good way, for example, to find the resultant of a complex set of loads on a beam or truss.

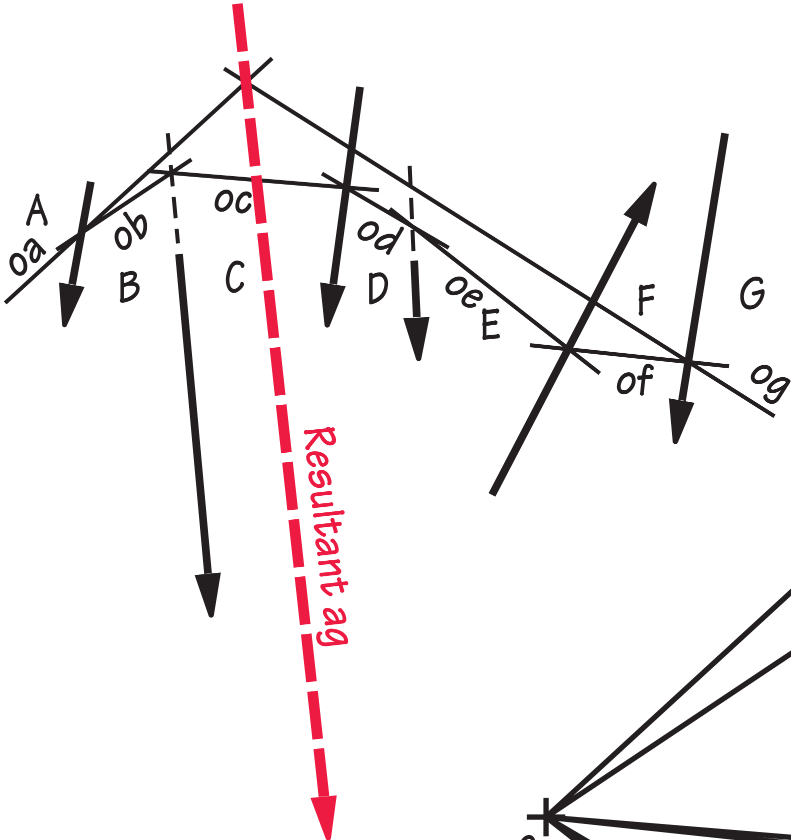
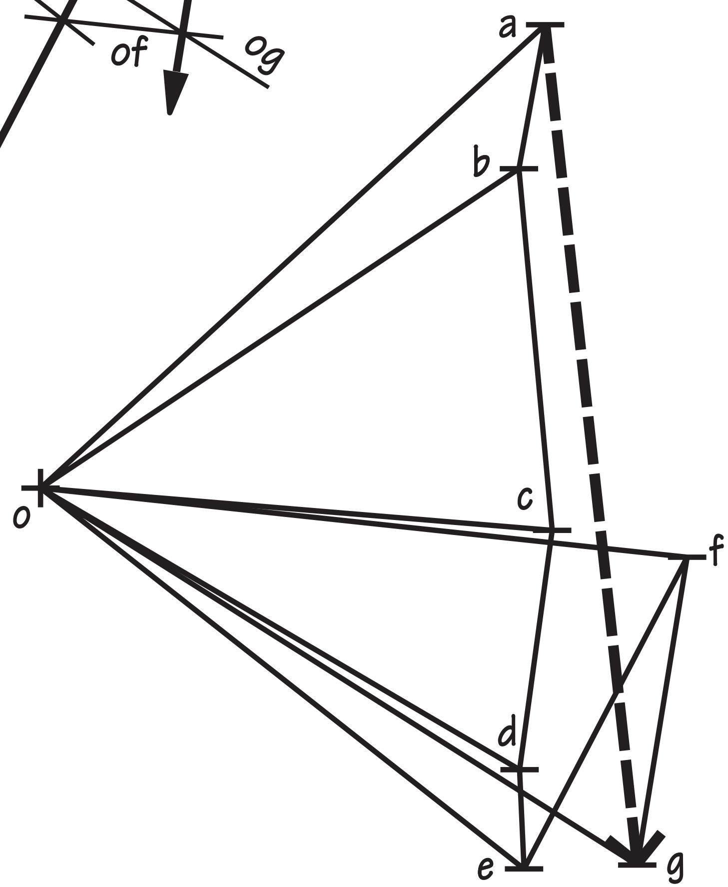


Diagram Of Force Vectors

1 MN



Force Polygon

1 MN

*Click on the **Contents** button to begin a new lesson.*

Click on the image of the Güell Chapel to return to the beginning of this lesson.