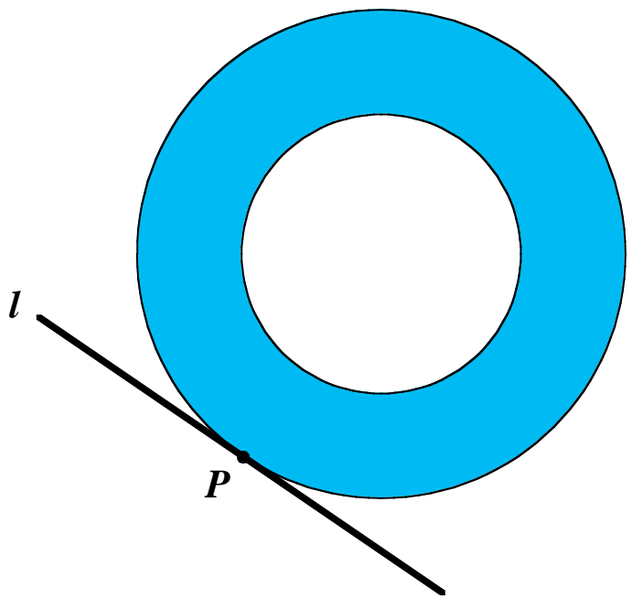
Important: **The term "construct" in all of the questions means "construct using an unmarked ruler and a compass". The phrase "unmarked ruler" stands for any ruler that may be used only as a straight edge to draw straight line segments. When you use a compass, show the (intermediate) circular arcs you draw in your constructions (do not erase them). Use words to describe BRIEFLY what you have done.**

[9] **1.** (a) Construct the center of the ring given below.

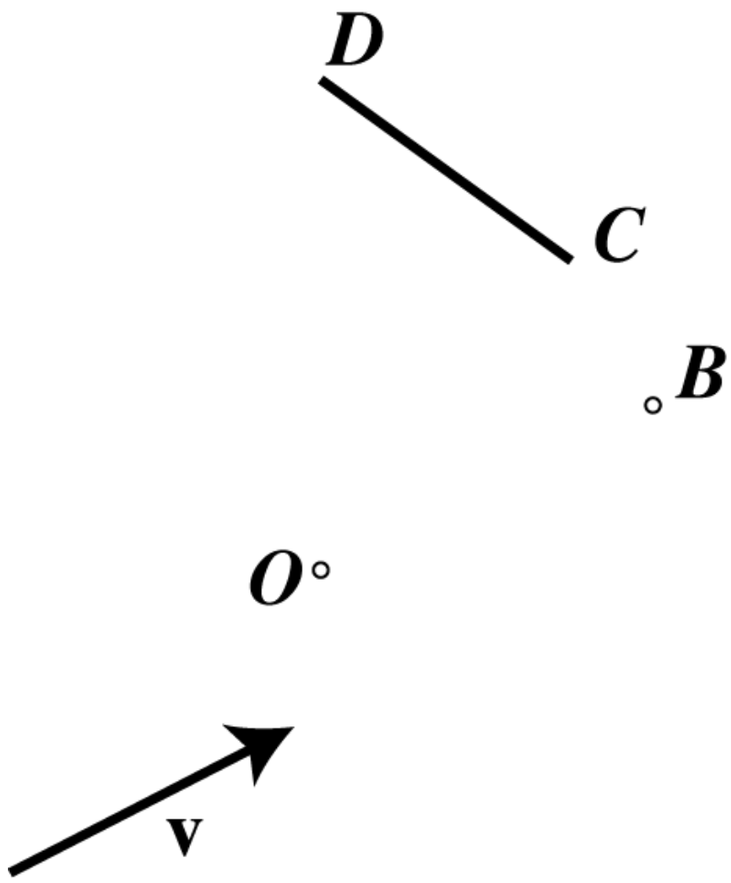
(b) Identify the point Q on the outer circle that is antipodal (opposite) to the given point *P*, and then construct the line that is parallel to the line *l* (as shown in the figure) and it touches the outer circle at a point Q.



[8] **2**. Construct an obtuse golden triangle so that the line segment shown below is the whole base of that triangle.

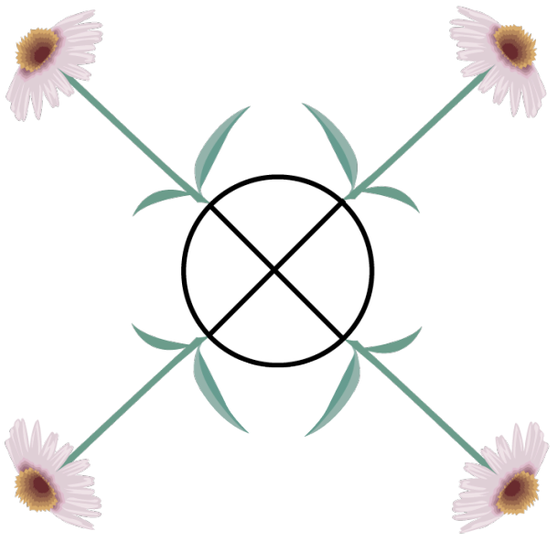
[9] **3. (a)** Construct the image of the line segment ***CD*** given below under the rotation $f = rot(O, 30°)$ centered at the given point ***O*** and through an angle of 30º. (Note: you would first need to construct an angle of 30º.)

(b) Construct the image of the point ***B*** under the composition of $g = rot(O, -30°)$ followed by the translation $h = tran_{\mathbf{v}}$ along the shown vector **v**.

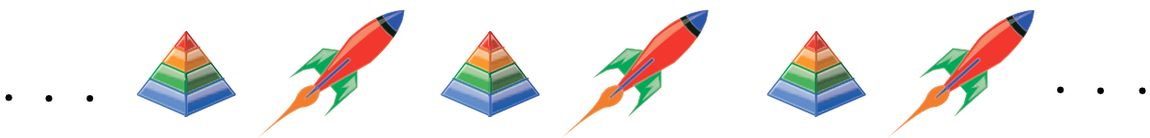


[12] 4.

(a) Find the group of the symmetries of the following object. Clearly identify the symmetries.



(b) Find the group of the symmetries of the following Frieze pattern. Clearly identify the symmetries.

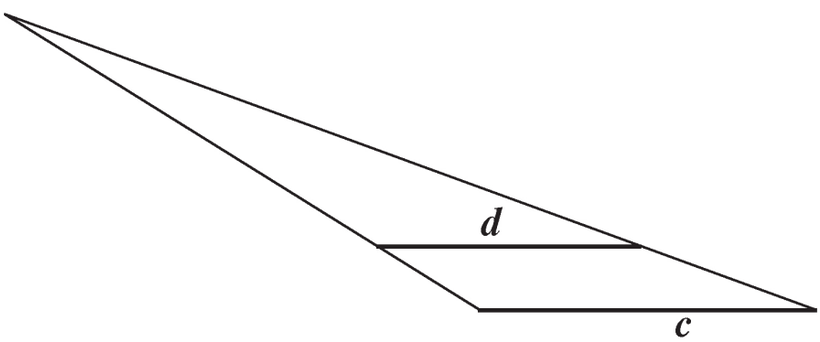


(c) Draw an example of a design having **exactly** five symmetries (including id). [Note: you only need to sketch such a design; you do not have to precisely construct it (which does not mean that you should make an ugly sketch)].

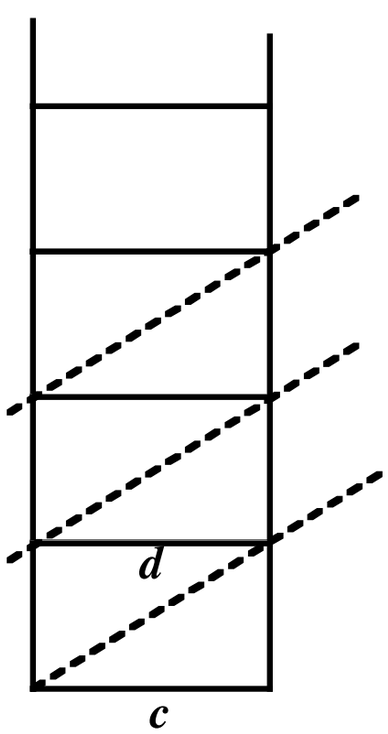
[9] **5.** The following picture shows a perspective scheme of a (horizontal) railroad track with the two parallel rails depicted by the two intersecting lines, and two consecutive parallel wooden railroad ties represented by the parallel line segments denoted by *c* and *d*. Assume that any two consecutive railroad ties are placed at the same fixed distance.

(a) Sketch the horizon line.

(b) Precisely draw two more railroad ties. (See the hint further down!)



Hint: the figure below shows the non-perspective image of the railroad (bird's eye view). To solve the above problem you may use the fact that the dotted lines shown below (along the diagonals of the rectangles) are parallel. [Note again: the figure below is to help you solve question 5; all of your construction should be shown above.



[8] **6.** In the two figures below (Figure 1 and Figure 2) we show the first two steps in the construction of a fractal. Note that the two smaller squares in the second figure have side twice shorter than the sides of the largest square.

(a) Draw the figure representing the next step in the construction of the fractal. You should do this directly in Figure 2 below.

(b) The final fractal **F** will be constructed after infinitely many steps (the first few of them are described in Figures 1, 2 and in the correct solution to question (a) here). Find a central similarity of stretching factor not equal to 1 that will send a part of the fractal **F** into itself. (To get full marks here, you need to indicate in the figure you draw in part (a) where the center of the central similarity is, and you need to state a specific number for the stretching factor of that central similarity.)

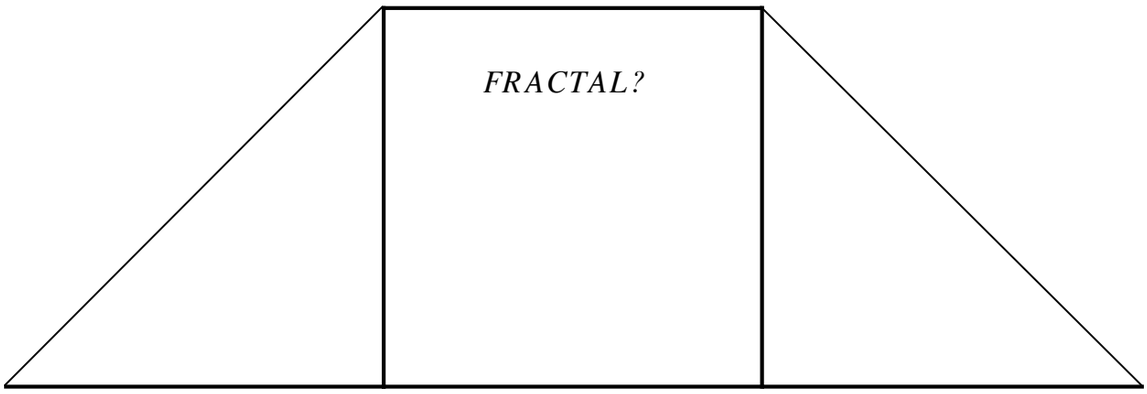


Figure 1.

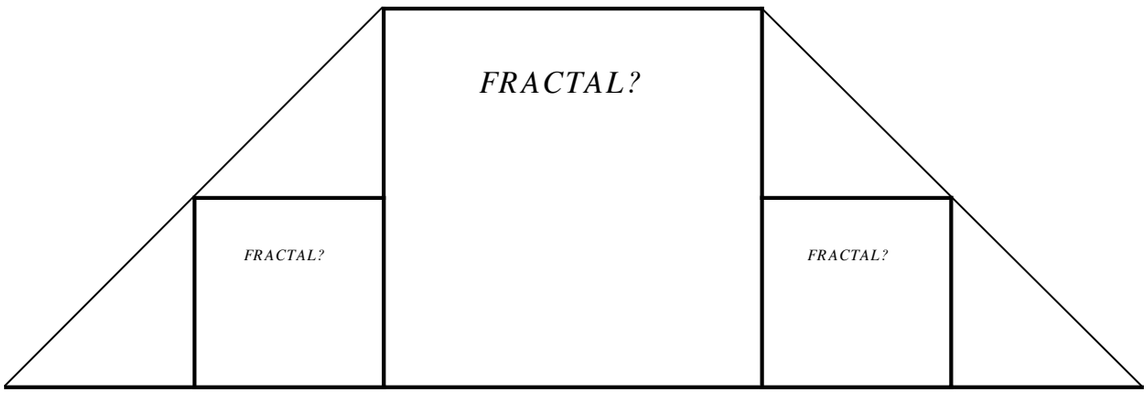
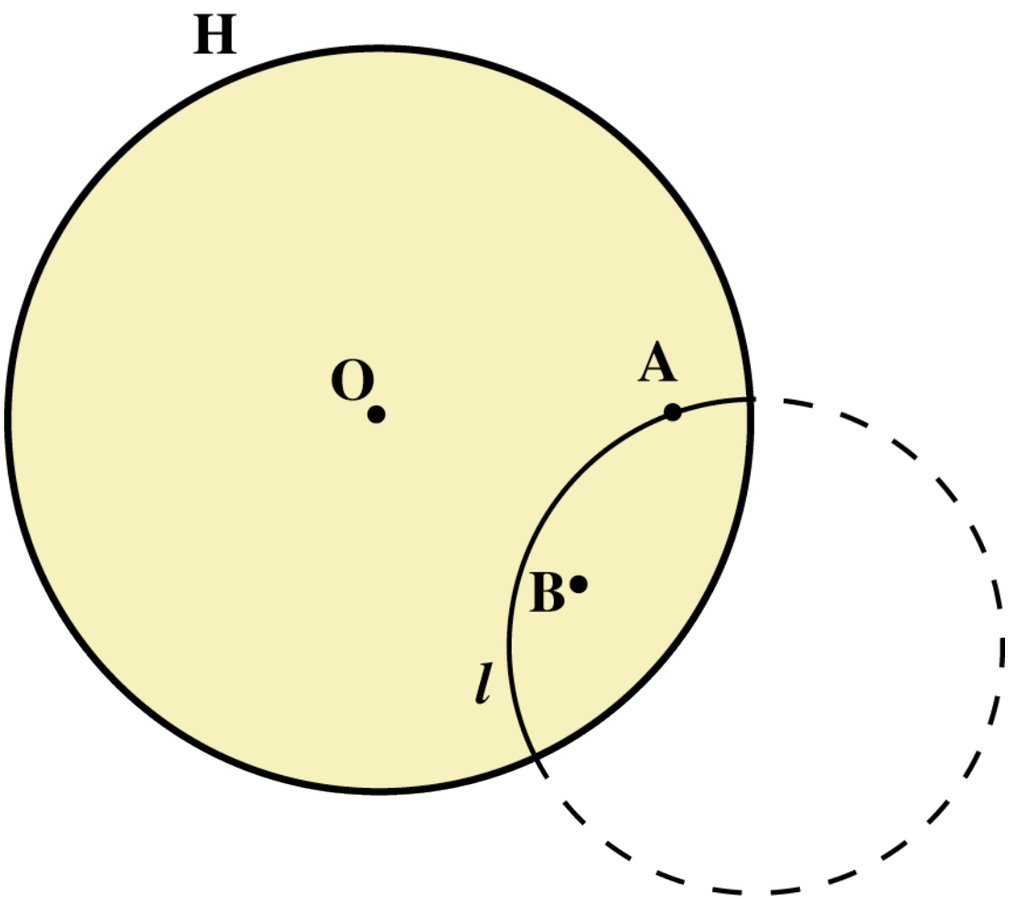


Figure 2.

[8] **7.** We are given a hyperbolic line *l*, a point *A* on that hyperbolic line, and a point *B* outside the line *l*.

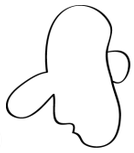
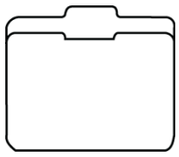
(a) Construct one hyperbolic line (label it h_1) parallel to *l* and passing through *B*, and one hyperbolic line (label it h_2) intersecting *l* and passing through *B*.

(b) Construct the hyperbolic line passing through both *A* and *B*.



[7] **8.** The objects depicted below consist of the black coloured points only.

(a) Which of the following four designs are mutually homotopic?

(1)  (2)  (3)  (4) 

(b) Show that the two designs shown below are homotopic by drawing at least three in-between sketches showing the left object can be continuously deformed into the right object.