UNIVERSITY OF MANITOBA

DATE: December 14, 2007, 2007 — FINAL EXAMINATION
PAPER # 487 — TITLE PAGE
COURSE: MATH 2400 — TIME: 3 hours
EXAMINATION: Graph Theory — EXAMINER: M. Davidson

FAMILY NAME: (Print in ink) _____

GIVEN NAME(S): (Print in ink) _____

STUDENT NUMBER: _____

SIGNATURE: (in ink) _____
(I understand that cheating is a serious offense)

INSTRUCTIONS TO STUDENTS:

This is a 3 hour exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 10 pages of questions. Please check that you have all the pages.

The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 48 points.

Answer questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	0	
2	0	
3	24	
4	24	
5	0	
6	0	
7	0	
8	0	
9	0	
Total:	48	

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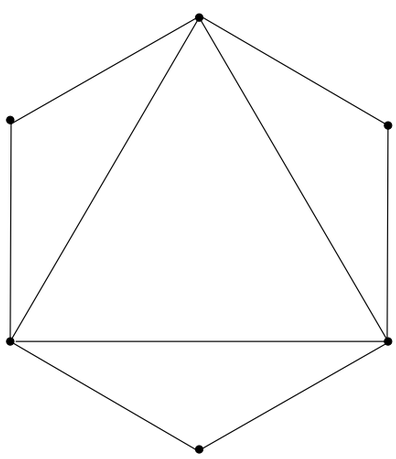
1. (a) Draw a graph on 5 vertices that is connected but not stongly connected.

(b) Draw a graph on 5 vertices that is stongly connected.

(c) Draw a graph on 5 vertices that is Eularian but not Hamiltonian.

(d) Draw a graph on 5 vertices that is Hamiltionian but not Eularian.

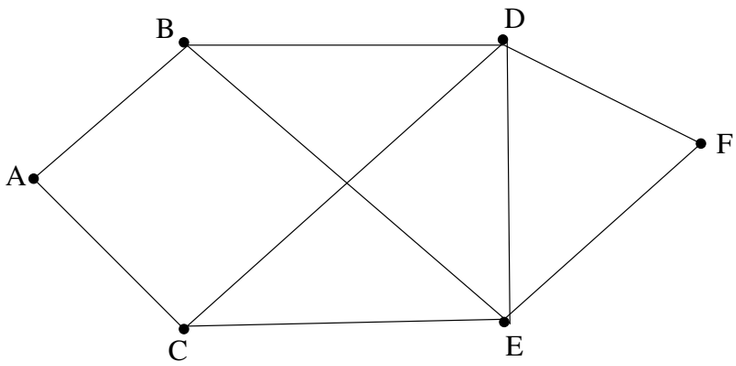
Let G be the following graph



(a) Draw the complement of G, $\overline{G}$.

(b) Are G and $\overline{G}$ isomorphic? (Give the appropriate labeling if they are isomorphic, or give a reason why they are not isomorphic)

2. Let G be the following graph:



(a) The number of vertices of G is _____.

(b) The number of edges of G is _____.

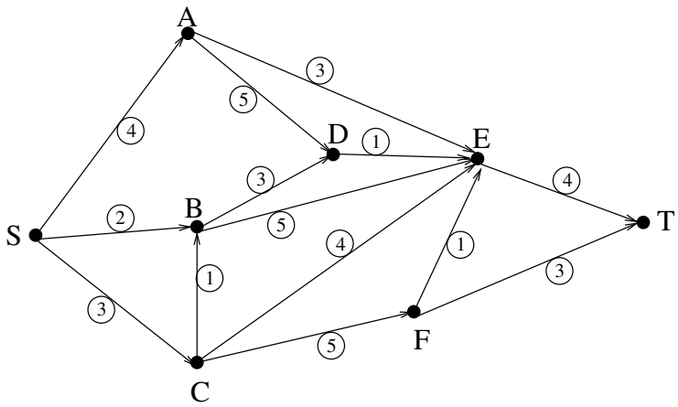
(c) The degree sequence of G is _____.

(d) Use the above to verify the handshaking lemma for G.

(e) Draw a subgraph (with labels) of G that is a complete graph.

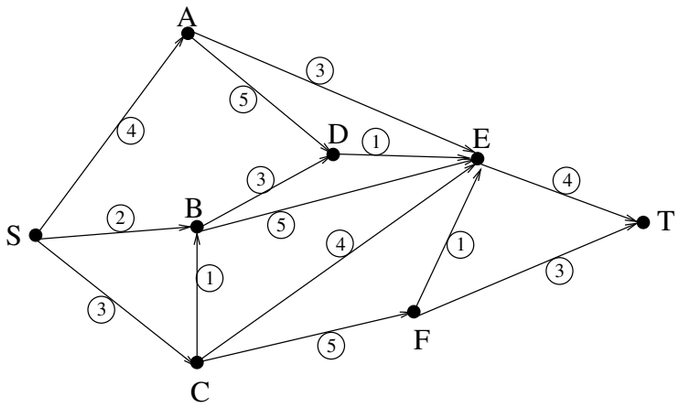
(f) Give the incident matrix of G.

[24] 3. Consider the graph:



Find the shortest path from S to T. Explain your method.

[24] 4. Again consider the graph:



(a) Find the longest path from S to T

(b) Complete the following table according to scheduling the events represented in the above graph:

E-Earliest start time L-Latest start time F-Float time

	SA	SB	SC	AD	AE	BD	BE	CB	CE	CF	DE	ET	FE	FT
E														
L														
F														

5. Consider the following results of a survey of snacks where the preferred choice is underlined:

Survey1	Survey2	Survey3
Apple pie - Bagel	Apple pie - Bagel	Apple pie - Bagel
Bagel - Cookie	Bagel - Cookie	Bagel - Cookie
Cookie - Donut	Cookie - Donut	Cookie - Donut
Donut - Eclair	Donut - Eclair	Donut - Eclair
Eclair - Apple pie	Eclair - Apple pie	Eclair - Apple pie
Apple pie - Cookie	Apple pie - Cookie	Apple pie - Cookie
Cookie - Eclair	Cookie - Eclair	Cookie - Eclair
Eclair - Bagel	Eclair - Bagel	Eclair - Bagel
Bagel - Donut	Bagel - Donut	Bagel - Donut
Donut - Apple pie	Donut - Apple pie	Donut - Apple pie

   (a) Define a tournament and verify that each of these surveys defines a tournament.

   (b) Draw a directed graph for each of the surveys.

(c) For each survey:

i. What is the score sequence?

ii. Is it transitive?

iii. Is it strongly connected?

(d) For each survey, if possible, find a ranking of the preferences.

(e) Which survey is the most consistent? Which is the least consistent?

6. Cost Cutter airlines currently maintains flights between the following cities: Athens (A) , Barcelona (B), Copenhagen (C), Detroit (D), Elie (E) and Frankfurt (F). They wish to eliminate as many flights as possible but still maintain connections with each city, with minimal overhead cost. Below is a table giving the overhead cost of maintaining a flight between any two of the cities:

	A	B	C	D	E	F
A	-	20	40	20	60	60
B	20	-	60	30	80	70
C	40	60	-	40	50	80
D	20	30	40	-	50	60
E	60	80	50	50	-	70
F	60	70	80	60	70	-

   (a) What flights should be kept to achieve this goal? (Explain the process you employed to find this answer)

7. (a) State Euler's formula for connected planar graphs.

(b) Use the above to show that if a simple connected planar graph G having n-vertices, m-edges, then $m \leq 3n - 6$

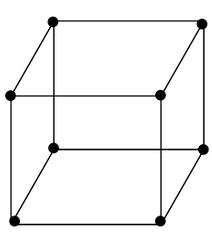
(c) Show that K_5 is not planar.

(d) If a connected planar graph graph had 5 vertices and 10 edges, how many faces would it have?

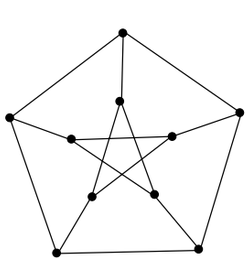
8. State Kuratowski's Theorem.

9. Decide whether each of the following graphs is planar or not. If it is planar, give a plane drawing, and find its corresponding dual graph. If it is not, give an expanation as to why not.

(a)



(b)



(c)