UNIVERSITY OF MANITOBA

DATE: April 12, 2010 — FINAL EXAMINATION
TITLE PAGE

COURSE: MATH 2400 — TIME: 3 hours

EXAMINATION: Graph Theory — EXAMINER: M. Davidson

FAMILY NAME: (Print in ink) _____

GIVEN NAME(S): (Print in ink) _____

STUDENT NUMBER: _____

SIGNATURE: (in ink) _____
(I understand that cheating is a serious offense)

INSTRUCTIONS TO STUDENTS:

This is a 3 hour exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 10 pages of questions. Please check that you have all the pages.

The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 24 points.

Answer questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	0	
2	0	
3	0	
4	24	
5	0	
6	0	
7	0	
8	0	
9	0	
10	0	
Total:	24	

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1. Draw a simple graph on 6 vertices satisfying the given conditions if one exists. If no such graph exists, explain why not.

   (a) 3 regular.

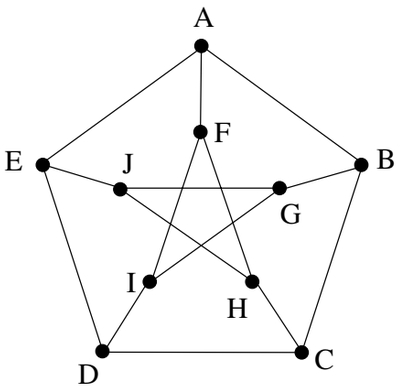
   (b) Ismorphic to its complement.

   (c) Hamiltonian but not Eularian nor semi-Eulerian.

   (d) Having vertex connectivity $\kappa = 1$ and edge connectivity $\lambda = 2$.

2. (a) Prove that a bipartite graph has no cycles of odd length.

(b) Show that the Peterson graph (below) is not bipartite.



Petersen graph

(c) What is the vertex connectivity of the Peterson graph? Can you find a vertex cutset with

i. 3 vertices?

ii. 4 vertices?

iii. 5 vertices?

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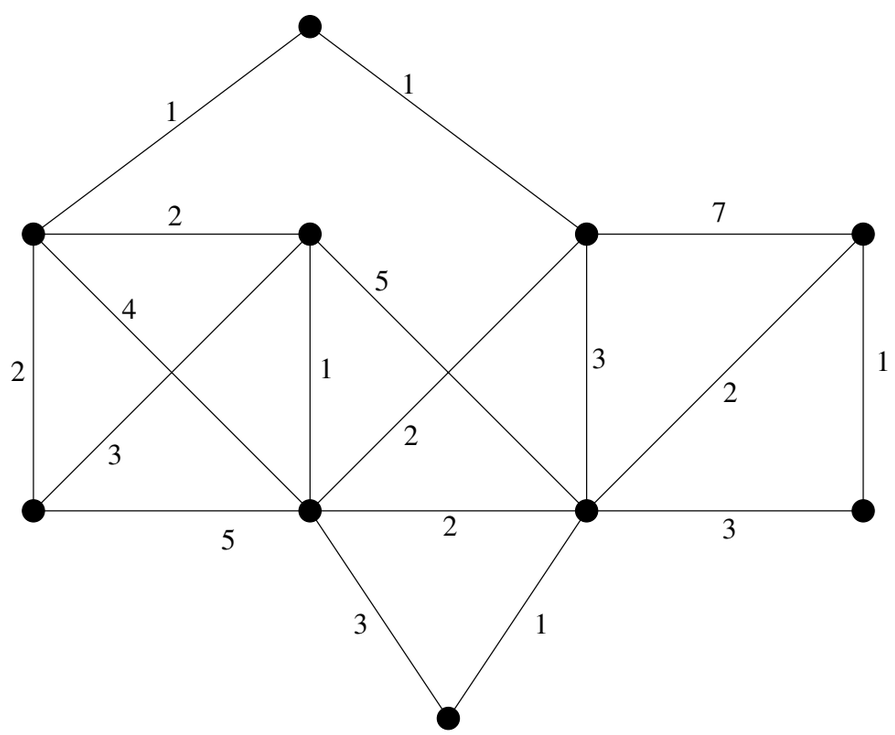
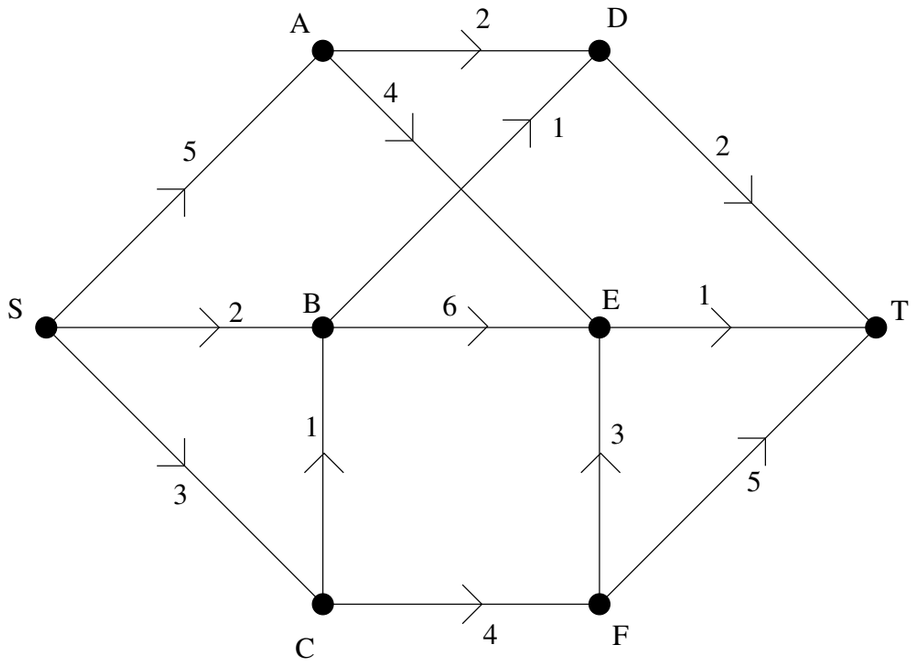
3. Solve the following Chinese postman problem. Show the intermideate steps of the shorest path algorithm used to solve the problem. (You may do so by table or by labels in the graph). Give the appropriate closed walk, and its weight.



[24] 4. In the following weighted digraph:



(a) Find the longest path from S to T

(b) Complete the following table according to scheduling the events represented in the above graph:

E-Earliest start time L-Latest start time F-Float time

	SA	SB	SC	AD	AE	BD	BE	CB	CF	DT	ET	FE	FT
E													
L													
F													

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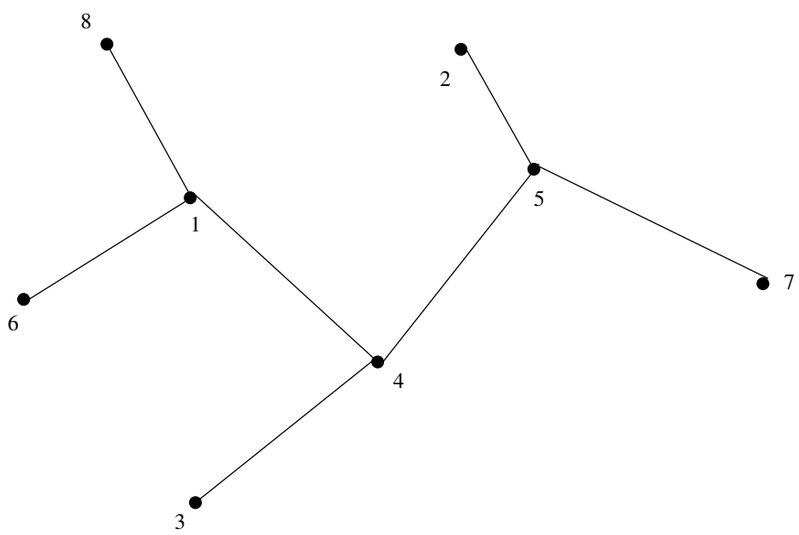
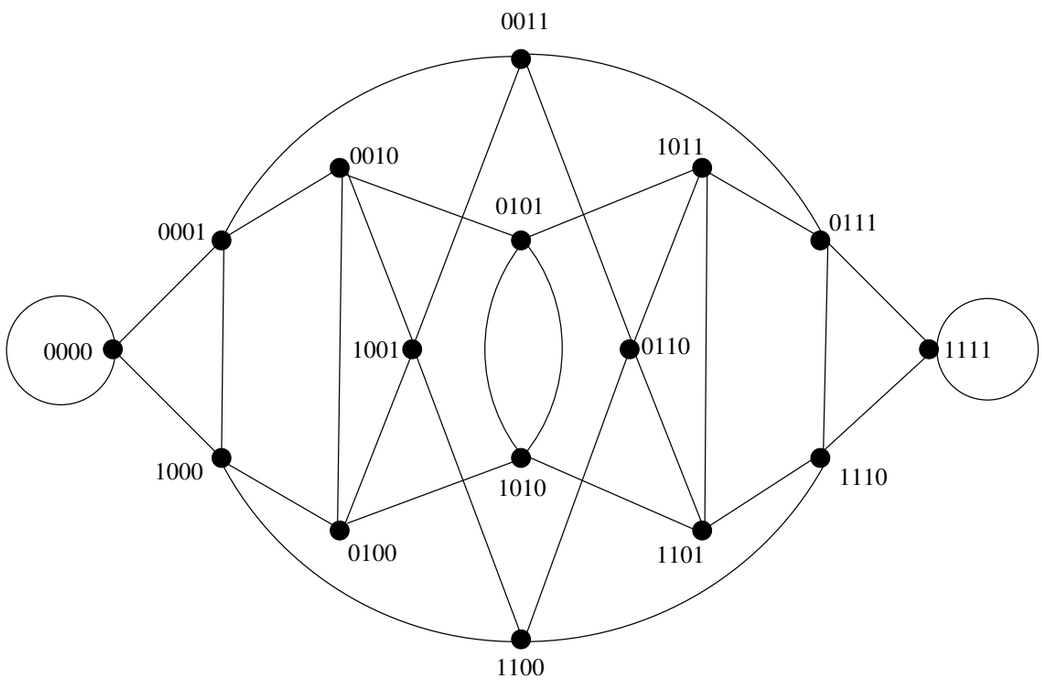
5. (a) Find the Prüfer sequence for the following labelled tree

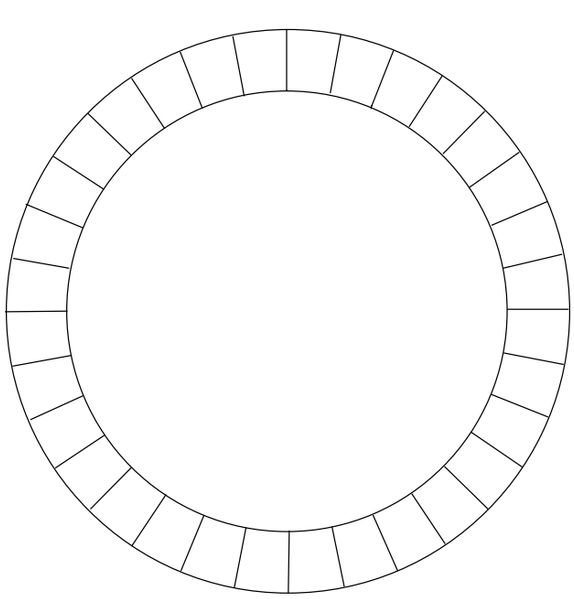


(b) Draw the tree have Prüfer sequence

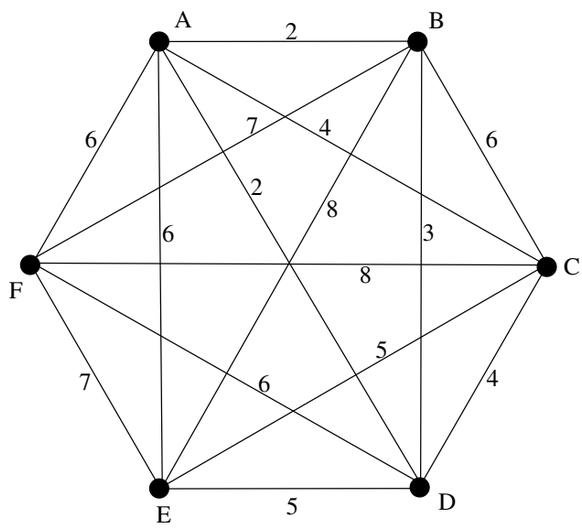
5 7 5 3 1 4

6. Below is the underlying graph of the digraph necessairy to solve the rotating drum problem for a drum with 32 divisions. Draw the appropriate digraph (with the arrow to indicate the direction of the arc, and the appropriate labels) Solve the rotating drum problem (put your answer in the drum below).





7. Consider the following weighted graph (K_6)



(a) Find a minimal spanning tree. Give its weight, and explain briefly how it was found.

(b) Find an upper bound for the solution to the travelling salesman problem using the heuristic algorithm starting at D.

(c) Find an lower bound for the solution to the travelling salesman problem by removing D.

8. (a) State Euler's formula for connected planar graphs.

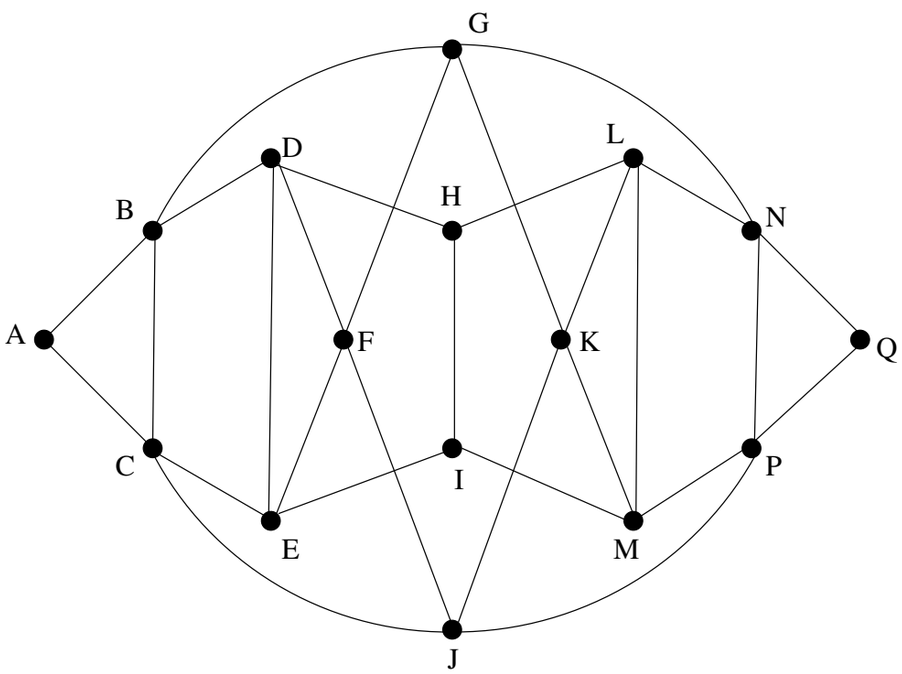
(b) Use the above to show that if a simple connected planar graph G having V-vertices, E-edges, and no triangles then $E \leq 3V - 6$

(c) Show that $K_{3,3}$ is not planar.

(d) If a connected planar graph graph had 6 vertices and 9 edges, how many faces would it have?

9. (a) State Kuratowski's Theorem.

(b) Use Kuratowski's Theorem to show that the following graph is not planar.



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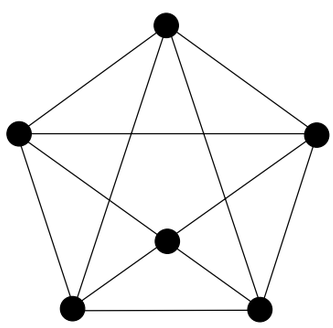
10. For the following planar graphs, give a plane drawing, and find its corresponding dual graph.

(a)



(b)