UNIVERSITY OF MANITOBA

DATE: March 3, 2010 — MIDTERM TITLE PAGE

COURSE: MATH 2400 — TIME: 50 minutes

EXAMINATION: Graph Theory — EXAMINER: M. Davidson

FAMILY NAME: (Print in ink) _____

GIVEN NAME(S): (Print in ink) _____

STUDENT NUMBER: _____

SIGNATURE: (in ink) _____

(I understand that cheating is a serious offense)

INSTRUCTIONS TO STUDENTS:

This is a 50 minute exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, 6 pages of questions. Please check that you have all the pages.

The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 100 points.

Answer questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	15	
2	20	
3	5	
4	15	
5	15	
6	15	
7	15	
Total:	100	

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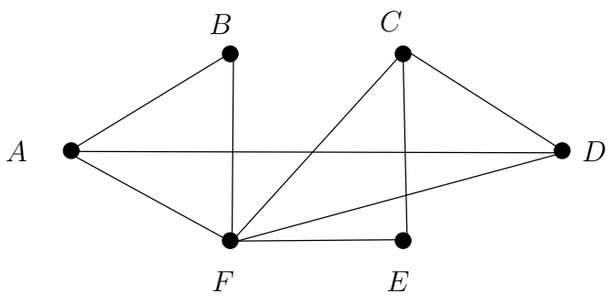
[15] 1. Consider the graph:



(a) What is the degree of each vertex of the graph?

vertex	A	B	C	D	E	F
degree						

(b) What is the degree sequence of the graph?

(c) State the Handshaking Lemma. Use it to find the number of edges in the graph.

(d) Is this graph Eulerian? If so, give a suitable trail. If not, explain why not.

(e) Is this graph Hamiltonian? If so, give a suitable cycle. If not, explain why not.

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[20] 2. Fill in the following table according to these directions:

In the column labeled *vertices*, write the number of vertices of the graph.

In the column labeled *edges*, write the number of edges of the graph.

In the column labeled *regular/degree*, if the graph is regular, write **yes** and then the degree of each vertex; otherwise just write **no**.

In the column labeled *Eulerian*, write **yes** if the graph is Eulerian, otherwise write **no**.

In the column labeled *Hamiltonian*, write **yes** if the graph is Hamiltonian, otherwise write **no**.

	vertices	edges	regular/degree	Eulerian	Hamiltonian
K_{13}					
K_{18}					
$K_{4,10}$					
$K_{5,9}$					
$K_{8,8}$					
N_{24}					
P_{17}					
C_{21}					

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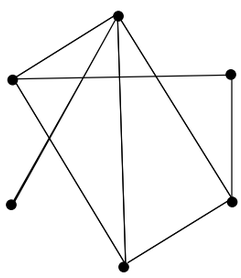
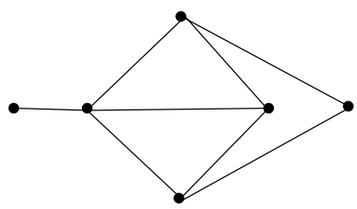
[5] 3. Show that the following graphs are isomorphic by giving the appropriate labeling, or give a reason why they are not isomorphic.

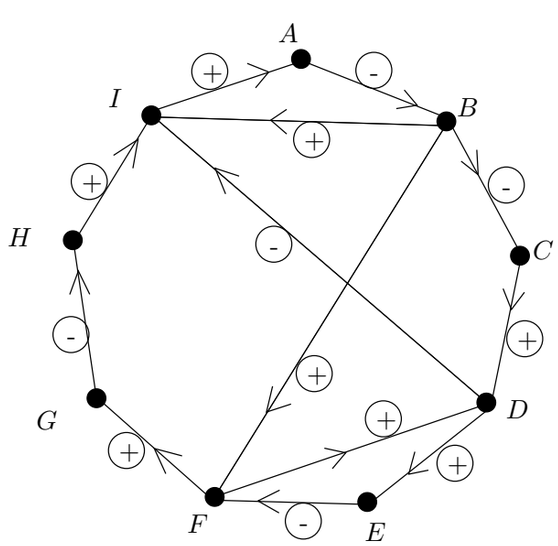
Graph A



Graph B



[15] 4. In the signed digraph below, find five (5) cycles starting at A, and decide if the cycle is a positive feedback cycle, or a negative feedback cycle.



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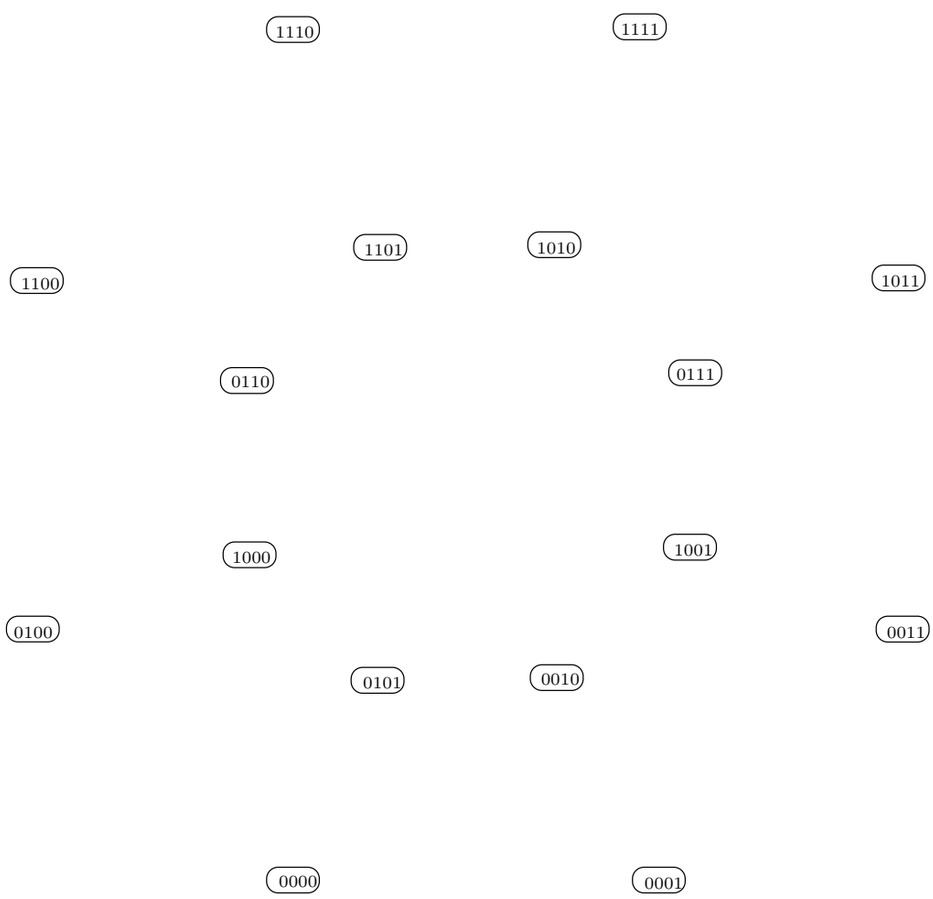
[15] 5. (a) On the labeled vertices below, draw all the edges in the cube graph Q_4.



(b) Find a Gray code of 4-digit binary words. Write the code below and indicate it in the graph above. (What is the Gray code with respect to the above graph?)

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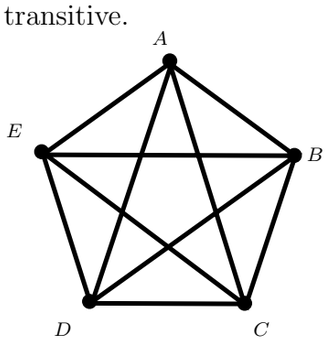
[15] 6. (a) Below are the underlying graphs of tournaments. Draw the arcs necessary to give a tournament that satisfies the description.

(b) Give the score sequence of the tournament.

(c) Give a ranking if one exists.

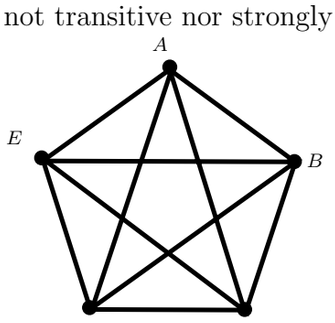
i. transitive.



Score sequence:

Ranking:

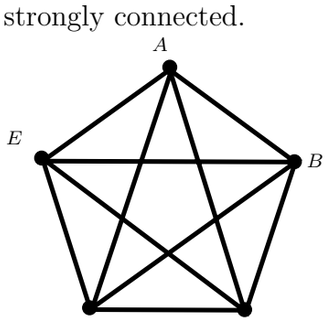
ii. not transitive nor strongly connected.



Score sequence:

Ranking:

iii. strongly connected.



Score sequence:

Ranking:

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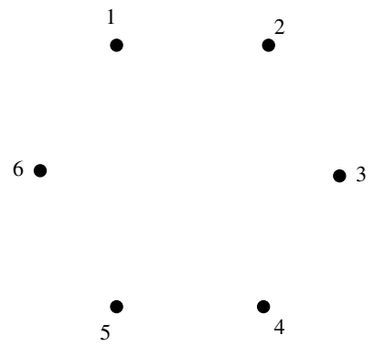
[15] 7. Let D be the digraph that has adjacency matrix A.

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(a) On the labeled vertices below, draw the digraph D.



(b) Given that $A^4 = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 2 & 0 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$, interpret the (3,4)-entry with respect to the digraph D.

(c) Give the incidence matrix of the digraph D. (Label the arcs in order of when they appear in the adjacency matrix.)