

Some formulas from Vector Calculus:

Green's Theorem:

$$\oint_C P dx + Q dy = \iint_R \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

Divergence Theorem:

$$\oiint_S \vec{F} \cdot \hat{n} dS = \iiint_V \nabla \cdot \vec{F} dV$$

Stokes's Theorem:

$$\oint_C \vec{F} \cdot d\vec{r} = \iint_S (\nabla \times \vec{F}) \cdot \hat{n} dS$$

The following are true for any interger  $n$ :

$$\sin(n\pi) = 0, \quad \cos(n\pi) = (-1)^n, \quad \sin\left(\frac{(2n+1)\pi}{2}\right) = (-1)^n, \quad \cos\left(\frac{(2n+1)\pi}{2}\right) = 0.$$

Some trigonometric formulas:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin A \cos B = \frac{1}{2}(\sin(A+B) + \sin(A-B))$$

$$\cos A \cos B = \frac{1}{2}(\cos(A+B) + \cos(A-B))$$

$$\sin A \sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$$

$$\cos^2 A = \frac{1}{2} + \frac{1}{2} \cos 2A$$

$$\sin^2 A = \frac{1}{2} - \frac{1}{2} \cos 2A$$

Some formulas from Fourier Series:

Let  $f(x)$  be defined and piece-wise smooth on  $\{x : 0 \leq x \leq 2L\}$ . The **(Full) Fourier series** of  $f(x)$  is given by :

$$f(x) \approx \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right)$$

$$a_n = \frac{1}{L} \int_c^{c+2L} f(x) \cos \frac{n\pi x}{L} dx$$

$$b_n = \frac{1}{L} \int_c^{c+2L} f(x) \sin \frac{n\pi x}{L} dx$$

Half-range expansions:

Let  $f(x)$  be defined and piece-wise smooth on  $\{x : 0 \leq x \leq L\}$ .

The **Fourier Cosine series** of  $f(x)$  is given by :

$$f(x) \approx \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{L} x$$

$$a_n = \frac{2}{L} \int_0^L f(x) \cos \frac{n\pi}{L} x dx$$

The **Fourier Sine series** of  $f(x)$  is given by :

$$f(x) \approx \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{L} x$$

$$b_n = \frac{2}{L} \int_0^L f(x) \sin \frac{n\pi}{L} x dx$$

Other expansions:

Let  $f(x)$  be defined and piece-wise smooth on  $\{x : 0 \leq x \leq L\}$ .

$$f(x) \approx \sum_{n=1}^{\infty} c_n \cos \frac{(2n-1)\pi}{2L} x$$

$$c_n = \frac{2}{L} \int_0^L f(x) \cos \frac{(2n-1)\pi}{2L} x dx$$

Let  $f(x)$  be defined and piece-wise smooth on  $\{x : 0 \leq x \leq L\}$ .

$$f(x) \approx \sum_{n=1}^{\infty} d_n \sin \frac{(2n-1)\pi}{2L} x$$

$$d_n = \frac{2}{L} \int_0^L f(x) \sin \frac{(2n-1)\pi}{2L} x dx$$

Some trigonometric integrals:

You may use any of the following formulas without further explanation:  
( $k$  is a non-zero constant)

$$\int \cos kx \, dx = \frac{1}{k} \sin kx$$

$$\int x \cos kx \, dx = \frac{1}{k^2} \cos kx + \frac{x}{k} \sin kx$$

$$\int x^2 \cos kx \, dx = \frac{x^2}{k} \sin kx + \frac{2x}{k^2} \cos kx - \frac{2}{k^3} \sin kx$$

$$\int \sin kx \, dx = -\frac{1}{k} \cos kx$$

$$\int x \sin kx \, dx = \frac{1}{k^2} \sin kx - \frac{x}{k} \cos kx$$

$$\int x^2 \sin kx \, dx = -\frac{x^2}{k} \cos kx + \frac{2x}{k^2} \sin kx + \frac{2}{k^3} \cos kx$$