Math 3132 **Test #1** **October 16, 2008**

Note: Simplify all answers. There are four (4) questions for a total of 40 marks. Time: 60 minutes.

Formulae: Green's Theorem: $\oint_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$.

Divergence Theorem: $\oiint_S \mathbf{F}\cdot\hat{\mathbf{n}}\,dS = \iiint_V \nabla\cdot\mathbf{F}\,dV$.

Values

(9) 1. (a) Let $\mathbf{F}(x,y,z) = (ye^z)\hat{\mathbf{i}} + (xe^z - 1)\hat{\mathbf{j}} + (xye^z + z)\hat{\mathbf{k}}$. Calculate $\nabla\times\mathbf{F}$, showing details.

(b) Find a function φ such that $\nabla\varphi = \mathbf{F}$.

(c) Evaluate the line integral $\int_C \mathbf{F}\cdot d\mathbf{r}$ where C is the straight line from (−1, 1, 0) to (1, 2, 1).

(11) 2. Consider the closed line integral $I = \oint_C x^2 y\,dx + y^2 x\,dy$ where C consists of the three straight line segments, first from (0, 0) to (1, 1), then from (1, 1) to (1, 0), and finally from (1, 0) to (0, 0).

(a) Evaluate the above integral I by ***parametrizing*** each part of C in terms of a parameter, t.

(b) Evaluate the above integral I by ***using Green's Theorem***.

(11) 3. Calculate the integral: $\iint_S \mathbf{F}\cdot\hat{\mathbf{n}}\,dS$, where $\mathbf{F} = (x^2)\hat{\mathbf{i}} + (2y)\hat{\mathbf{j}} + (z)\hat{\mathbf{k}}$, S is that part of the surface $z = y^2$ that lies inside the first octant and is bounded by the vertical surfaces $x = 0$, $y = 0$ and $x + y = 2$, and $\hat{\mathbf{n}}$ is the *upward* pointing unit normal to the surface S.
Remark: S consists of only one piece; it is part of $z = y^2$ as described above.

(9) 4. Let S be the *closed* surface enclosing the volume bounded by the upper hemisphere $x^2 + y^2 + z^2 = 4$ with $z \ge 0$ and the x-y plane.
Calculate the closed surface integral: $\oiint_S \mathbf{F}\cdot\hat{\mathbf{n}}\,dS$, where $\mathbf{F} = (z^3)\hat{\mathbf{i}} + (x^2)\hat{\mathbf{j}} + (z^2+4)\hat{\mathbf{k}}$, and $\hat{\mathbf{n}}$ is the unit *inner* normal to the surface S.

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