Platonic Solids

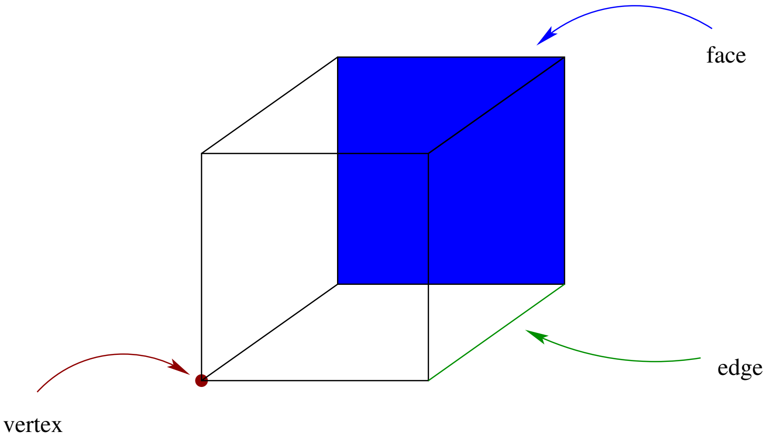
A convex polyhedron is *regular* if all of the bounding polygons are congruent regular polygons and if each vertex is adjacent to the same number of bounding polygons.

Platonic Solids

A regular convex polyhedron is known as a *Platonic Solid*.

There are 5 Platonic solids.

parts of polyhedra



Platonic solid: Tetrahedron

A tetrahedron has 4 faces which are equilateral triangles.

It has 4 vertices (each touching 3 faces).

It has 6 edges.

Platonic solid: Cube

A cube has 6 faces which are squares.

It has 8 vertices (each touching 3 faces).

It has 12 edges.

Platonic solid: Octahedron

An octahedron has 8 faces which are equilateral triangles.

It has 6 vertices (each touching 4 faces).

It has 12 edges.

Platonic solid: Dodecahedron

An dodecahedron has 12 faces which are regular pentagons.

It has 20 vertices (each touching 3 faces).

It has 30 edges.

Platonic solid: Icosahedron

An icosahedron has 20 faces which are equilateral triangles.

It has 12 vertices (each touching 5 faces).

It has 30 edges.

Euler Characteristic

If F is the number of Faces of a polyhedron, E is the number of Edges and V is the number of vertices, then the value of

$$F - E + V$$

is called the *Euler characteristic* of the polyhedron.

A look at the Euler characteristic of Platonic solids

Solid	Faces	Edges	Vertices	Euler characteristic
tetrahedron				
cube				
octahedron				
dodecahedron				
icosahedron				

Euler Characteristic

The Euler Characteristic of all convex polyhedra is 2.

SemiRegular Polyhedra

A convex polyhedron is *semiregular* if all of the bounding polygons are regular polygons (possibly more than one type) with edges the same length and if each vertex is adjacent to the same number of bounding polygons, and there exists a fixed cyclic orderof the types of polygons around all the vertices.

Archimedean Solids

There are 13 Archimedean solid which are semiregular polyhedra.

All prisms and antiprisms are semiregular polyhedra.

Why are there only 5 Platonic solids

$$F - E + V$$

Let n be the number of sides of a face.

Then $nF = 2E$, so $F = \frac{2E}{n}$.

Let m be the number of faces meeting at a vertex.

Then $mV = 2E$, so $M = \frac{2E}{m}$.

Substituting into the above we get

$$\frac{2E}{n} - E + \frac{2E}{m} = 2$$

Dividing by $2E$ we get :

$$\frac{1}{n} - \frac{1}{2} + \frac{1}{m} = \frac{1}{E}$$

We now look at the equation :

$$\frac{1}{n} + \frac{1}{m} = \frac{1}{2} + \frac{1}{E}$$

example question

A rhombicuboctahedron is an Archimedean solid. It has 24 vertices, each which meets 3 squares and one triangle. How many faces does it have? How many edges does it have?

example question

A truncated icosidodecahedron is an Archimedean solid. It has 30 square faces, 20 hexagonal faces and 12 decagonal faces.
How many vertices does it have? How many edges does it have?

Make your own polyhera models

Check out the following website:

`http://www.korthalsaltes.com/`