MATH 2500 Assignment #3

Due: October 31, 2012, Before Class (9:30)

Reminder: all assignments *must* be accompanied by an honesty declaration available on my website.

1. Find the least residue of the given expression in the given modulus
   (a) $222 \cdot 221 \cdots 109 \cdot 108 \cdot 106 \cdot 105 \cdots 2 \cdot 1$ in $\pmod{223}$
   (b) 13^{1138} in $\pmod{227}$
   (c) $892 \cdot 891 \cdots 107 \cdot 106 \cdot 104 \cdot 103 \cdots 2 \cdot 1$ in $\pmod{893}$
   (d) 12^{5043} in $\pmod{1001}$
   (e) 5^{70699} in $\pmod{6427}$
   (f) $466 \cdot 465 \cdots 9 \cdot 8 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ in $\pmod{467}$
   (g) 7^{11045} in $\pmod{2867}$
2. For each of the following, find $d(n)$ and $\sigma(n)$
   (a) 35 880
   (b) 5 852
   (c) 273 798
   (d) 20 020.
3. Find the first 5 (if they exist) elements of the aliquot sequence of the following:
   (a) 5 564
   (b) 1 305 184 $(= 2^5 \times 40\,787)$
   (c) 6 427
   (d) 864
4. Show that:
   (a) 8 128 is perfect.
   (b) 672 is 3-perfect.
   (c) 32 760 is 4-perfect.
   (d) 6232 is one of an amicable pair, and find the other number in the pair.
5. Suppose n is an odd perfect number such that $(n, 3) = 1$, show that $6n$ is a k-perfect number, and find k.