MATH 2500 Assignment #4

Due: November 28, 2012, Before Class (9:30)

Reminder: all assignments *must* be accompanied by an honesty declaration available on my website.

1. (a) How many primitive roots does 53 have?
   (b) What are the possible orders of an element modulo 53?
   (c) Show that 2 is a primitive root of 53.
   (d) Find 7 other primitive roots. (Note, these should be in least residue.)
   (e) What is the order of 16 (mod 53)? Of 32 (mod 53)? Of $11(\equiv 64)$ (mod 53)?
2. Which of the following quadratic congruences have solutions? (You do NOT need to find solutions if they exist.)
   (a) $x^2 + 9x + 13 \equiv 0 \pmod{43}$
   (b) $9x^2 + 7x + 6 \equiv 0 \pmod{73}$
   (c) $17x^2 + 11x + 1 \equiv 0 \pmod{67}$
3. Solve the following Legendre symbols:
   (a) $\left(\frac{4158}{6421}\right)$
   (b) $\left(\frac{17924}{8963}\right)$
   (c) $\left(\frac{3371}{9043}\right)$
4. Which of $2, 3, 4, 5$ are primitive roots of 3467?
   [Hint: Consider Euler's Criterion.]
5. (a) Show that if g and h are primitive roots of an odd prime p, and $g = h^k$, then k is odd.
   (b) Show that if g and h are primitive roots of an odd prime p then gh is *not* a primitive root.