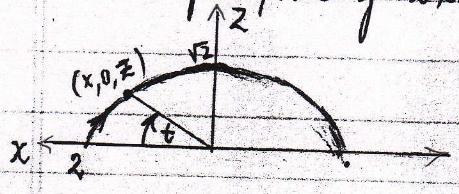
Answers to Math 2130 Test 1 (Winter) 2007

by Dawit yohannes
plankion@yahoo.Com

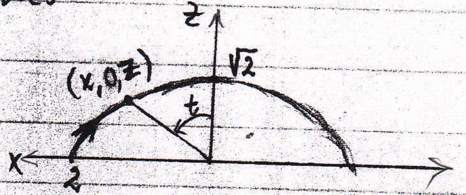
1) $9x-11y-3z-56=0$, or $-9x+11y+3z+56=0$

2) $\frac{7}{\sqrt{17}} = \frac{7\sqrt{17}}{17}$

3) projection of the Curve of intersection as Seen from the postive y-axis looks



or



$x = 2\cos t$
$y = 10 - 2\cos t - 2\sqrt{2}\sin t$
$z = \sqrt{2}\sin t$

$x = -2\sin t$
$y = 10 + 2\sin t - 2\sqrt{2}\cos t$; $0 \le t \le 2\pi$ (for both cases)
$z = \sqrt{2}\cos t$

4) $\frac{1}{\sqrt{5121}}\left(-32\hat{i} + \hat{j} + 64\hat{k}\right)$

5) $\frac{16}{57}y^2 + \frac{64}{57}\left(z-\frac{3}{8}\right)^2 = 1$ or $16y^2 + 64\left(z-\frac{3}{8}\right)^2 = 57$

ellipse

6) $\frac{t^3}{3}\hat{i} + \frac{1}{2}\ln|2t-1|\,\hat{j} + \frac{1}{4}e^{4t}\,\hat{k} + \vec{C}$

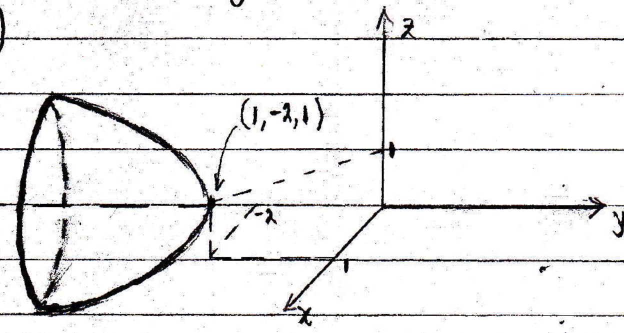
($\vec{C}$ is an arbitrary Constant Vector)

Answers by Dawit: ydawit@yahoo.com

Answers to Math 2130, February 7, 2008 Test I

1. $\frac{2\sqrt{22}}{11}$ or $\frac{\sqrt{88}}{11}$ or $\frac{\sqrt{8}}{\sqrt{11}}$

2. $(x-1)^2+(z-1)^2 = -(y+2)$ where $y \le -2$
(elliptic paraboloid)



3. $x = t$
$y = 1+t$; t is the parameter. $-\infty < t < \infty$
$z = 1-t-t^2$

4. a) $P(-3,-5,-6)$ is the intersection point.
Since $\vec{N}\cdot\vec{V} \neq 0$, $(2,2,1)\cdot(2,1,5) = 11 \neq 0$ (line and plane will intersect.
$\vec{N}$: Normal Vector to plane, $\vec{N} = (2,2,1)$
$\vec{V}$: Vector parallel to the line $\vec{V} = (2,1,5)$

b) $-9x+8y+2z-1=0$ or $9x-8y-2z+1=0$

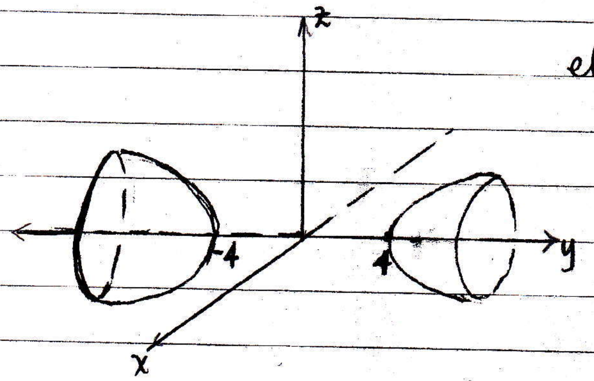
5. (a) $\hat{T}(0) = \frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{k} = \frac{1}{\sqrt{2}}(\hat{i}-\hat{k})$ (b) $\sqrt{2}\left(1-e^{-2\pi}\right)$ units

6. $\int(\vec{u}\times\vec{v})\,dt = -\frac{3}{4}t^2(t^2+2)\hat{i} + \frac{t^3}{4}(3t-8)\hat{j} + \frac{3t^2}{5}(t^3+5) + \vec{C}$

Where $\vec{C} = C_1\hat{i} + C_2\hat{j} + C_3\hat{k}$ is a Constant vector.

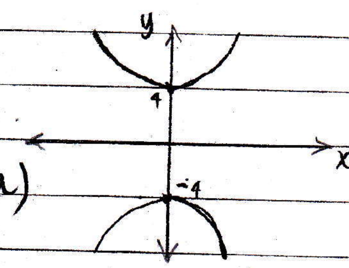
Answers to Math 2130 Feb 12, 2009 Test 1
by Dawit Yohannes (plankion@yahoo.com)

1) $x^2 + z^2 = y^2 - 16$, $|y| \geq 4 \Rightarrow y \geq 4, y \leq -4$



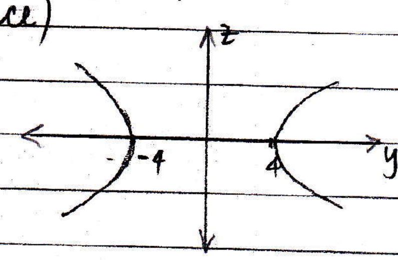
elliptic hyperboloid of two sheets

Traces: on the xy-plane ($z=0$): $y^2 - x^2 = 16$
(hyperbola)



on the xz-plane ($y=0$): $x^2 + z^2 = -16$ (not possible)
(no trace)

on yz-plane ($x=0$): $y^2 - z^2 = 16$
(hyperbola)



2) a) $60^\circ = \pi/3$ rad b) Area $= 6\sqrt{3}$

3) a) Show $\vec{N} \cdot \vec{V} = 0$ i.e, $(3,1,-4)\cdot(7,-1,5) = 0$

b) $d = 21/\sqrt{26} = \frac{21\sqrt{26}}{26}$

4) a) $\vec{r}(t) = (3 + 4\cos t)\hat{i} + (1 + 2\sin t)\hat{j} + (24\cos t + 16\sin t + 29)\hat{k}$; $0 \leq t \leq 2\pi$

b) $\hat{T}(\pi/2) = -\frac{1}{\sqrt{37}}(\hat{i} + 6\hat{k})$

5) a) $L = 6$ b) $\hat{T}(t) = \frac{1}{3}\left(-2\sin 2t\,\hat{i} + 2\cos 2t\,\hat{j} + \sqrt{5}\,\hat{k}\right)$

$\hat{N}(t) = -(\cos 2t\,\hat{i} + \sin 2t\,\hat{j})$

Answers to Math 2130 Test 1, 2009 (Fall) by Dawit yohannes
plankion@yahoo.Com

1. $x - 3y + 5z - 4 = 0$

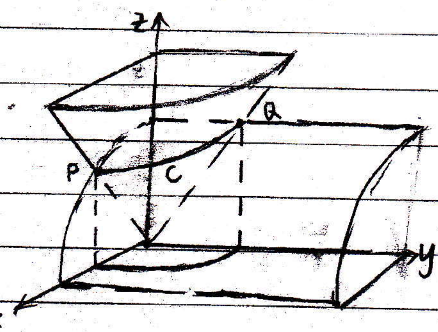
2. $d = \frac{18}{\sqrt{62}} = \frac{9\sqrt{62}}{31}$

3. $x = 1 + 2\cos t$
   $y = -\sqrt{5}\sin t$ for $0 \le t \le 2\pi$
   $z = 1 + 2\cos t - \sqrt{5}\sin t$

4. $\hat{T}(1) = \frac{5}{\sqrt{89}}\hat{j} - \frac{8}{\sqrt{89}}\hat{k}$

5 a) on the xz plane : $P\left(\frac{1}{\sqrt{2}}, 0, \sqrt{2}\right)$

on the yz plane : $Q(0, 2, 2)$

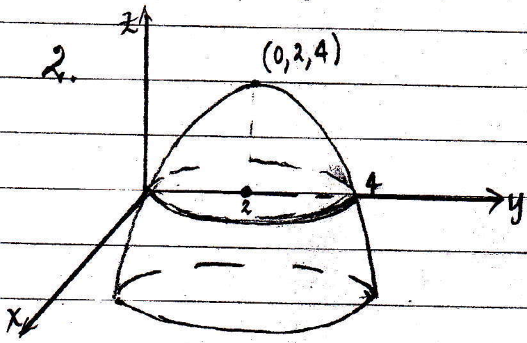


b) $z = 0$, $8x^2 + y^2 = 4$ (ellipse)

Answers to Math 2130 Feb. 11/2010 Test I

(by Dawit Yohannes, (plankton@yahoo.com))

1. $34x+16y-7z-119=0$

2. 

$x^2+(y-2)^2 = 4-z \quad ; \quad z \le 4$

: inverted elliptic paraboloid with the tip @ $(0,2,4)$.

3. $d = \dfrac{4\sqrt{6}}{\sqrt{5}} = \dfrac{4}{5}\sqrt{30}$

4. $\left.\begin{aligned} x &= 2\cos t \\ y &= 1-3\sin t \\ z &= 2 \end{aligned}\right\} \quad 0 \le t \le 2\pi$

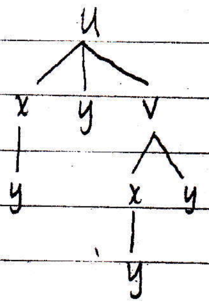
5. $L = \int_0^1 \sqrt{e^{2t}+e^{-2t}+2}\,dt = \int_0^1 \sqrt{(e^t+e^{-t})^2}\,dt = \int_0^1 (e^t+e^{-t})\,dt$

$= e - 1/e = \dfrac{e^2-1}{e}$

6. $\vec{T}(\pi/2) = 10(\hat{i}+\hat{k}) = 10\hat{i}+10\hat{k}$

plankion@yahoo.Com.

Answers to Math 2130 TEST 2: Dec. 2007 by Dawit Yohannes

1) 

$$\frac{du}{dy} = \frac{\partial u}{\partial v}\frac{\partial v}{\partial x}\frac{dx}{dy} + \frac{\partial u}{\partial v}\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x}\frac{dx}{dy} + \frac{\partial u}{\partial y}\Big)_{x,v}$$

2) limit does not exist

: along the x-axis $(y=0)$, limit is $\frac{1}{2}$

: along the y-axis $(x=0)$, limit is 2

→ : you can Sub. $y = mx$ to get $\frac{1+2m^2}{2+m+m^2}$; that Shows dependence on approach direction

3) $D_{\vec{N}} f = \pm \nabla f\Big|_P \cdot \frac{\vec{N}}{|\vec{N}|} = \pm \frac{63}{\sqrt{194}} = \pm \frac{63\sqrt{194}}{194}$

4) $(4x - z^3 - 2z) \Big/ (3xz^2 + 2x + 1)$

5) Critical points: $(6,-1)$, $(-6,1)$

both of them yield a Saddle point.

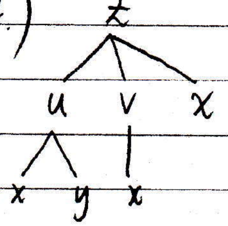
plankion@yahoo.Com

Answers to Math 2130 March 13/2008 Test 2

by dawit yohannes

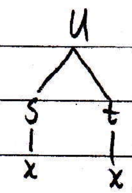
1) a) limit doesnot exist. (try to sub. $y = mx$ and show limit dependence on m (approach))

b) 5 (hint: try factorization and Cancellation)

2) 

(attention)

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\frac{dv}{dx} + \left.\frac{\partial z}{\partial x}\right)_{u,v}$$

3) 

$$\frac{du}{dx} = \frac{\partial u}{\partial s}\frac{ds}{dx} + \frac{\partial u}{\partial t}\frac{dt}{dx}$$

$$= e^{s+t}\left(\frac{2xs^3}{e^s - 3s^2x^2}\right) + \left(\sin t - e^{s+t}\right)\left(\frac{3x+2}{x^2+x}\right)t$$

4) $-\frac{3}{\sqrt{6}} = -\frac{\sqrt{6}}{2}$

5) critical points: (0,0), (1,-1), (1,1)

(0,0) yields relative-min

(1,-1) yields Saddle point

(1,1) yields Saddle point

by Dawit yohannes : plankion@yahoo.com

Answers to Math 2130 Test 2 March 12/2009

1. a) DNE (doesn't exist) b) 2 c) 0

2. $\frac{\partial u}{\partial y}\frac{\partial v}{\partial x} + \frac{\partial u}{\partial v}\frac{\partial v}{\partial z}\frac{dz}{dx} + \frac{\partial u}{\partial w}\frac{\partial w}{\partial x} + \left.\frac{\partial u}{\partial x}\right)_{v,w}$

3. $-\dfrac{2sy\sin s^2(x^3s+t^2)+(\cos s^2 - x)(x^3y+2st)}{2sy\sin s^2(s^2+2yt)+xe^t(x^3y+2st)}$

4. $-\frac{7}{\sqrt{5}} = -\frac{7\sqrt{5}}{5}$

5. $x = -1+t$, $y = 1+7t$, $z = 2+4t$; or $x+1 = \frac{y-1}{7} = \frac{z-2}{4}$

6. Critical points : $(0,\sqrt{2}), (0,-\sqrt{2}), (-\tfrac{2}{3},0), (\tfrac{2}{3},0)$

@ $(0,-\sqrt{2})$ and $(0,\sqrt{2}) \rightarrow$ Saddle point

@ $(-\tfrac{2}{3},0) \rightarrow$ rel. Min @ $(+\tfrac{2}{3},0) \rightarrow$ rel. Max.

by Dawit Yohannes
plankwon@yahoo.com

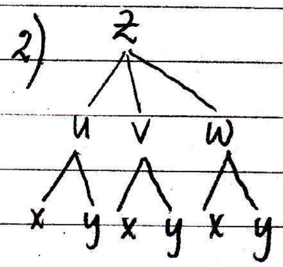
Answers to Math 2130 Test 2, 2009 (Fall) (Berry and Grim)

1) limit doesnot exist
try to Sub. $y = mx^2$ to get a result of the limit as

$\dfrac{2+3m^2}{3-2m+m^2}$ whose value depends on m.

→ along the x-axis $(y=0)(m=0)$ the limit is $2/3$

→ along the y-axis $(x=0)(m\to\infty)$ the limit is 3.

2) 

$$\left.\frac{\partial z}{\partial x}\right)_y = \left.\frac{\partial z}{\partial u}\right)_{v,w}\left.\frac{\partial u}{\partial x}\right)_y + \left.\frac{\partial z}{\partial v}\right)_{u,v}\left.\frac{\partial v}{\partial x}\right)_y + \left.\frac{\partial z}{\partial w}\right)_{u,v}\left.\frac{\partial w}{\partial x}\right)_y$$

$$= (2u+vw)(2)\left(\frac{x+y}{x-2y}\right)\left[\frac{(x-2y)(1)-(x+y)(1)}{(x-2y)^2}\right]$$

$$+(-4v+uw)\left[e^{3x}+xe^{3x}\cdot 3\right]+uv\sec^2(2x+4y)\cdot 2$$

3) $D_{\vec{N}} f\big|_p = \nabla f\big|_p \cdot \hat{N} = (0,2,-27)\cdot\dfrac{(-3,0,8)}{\sqrt{73}} = \dfrac{-8(27)}{\sqrt{73}} = \dfrac{-216}{\sqrt{73}}$

4) $\dfrac{\partial x}{\partial v} = \dfrac{2y-3xy^2+u(x-u)}{(3x+y)(2y-3xy^2)+y^3(x-u)}$

5) CPs : $(0,y)$ - all points along the y-axis,
$(x,0)$ - all points along the x-axis and
all points on the curve $xy = 2/3$

by Dawit Yohannes
plankion@yahoo.Com

Answers to Math 2130 Test 2, March 11, 2010

1) $$\frac{dz}{dx} = \left.\frac{\partial z}{\partial r}\right)_{s,y}\left[\left.\frac{\partial r}{\partial x}\right)_y + \left.\frac{\partial r}{\partial y}\right)_x \frac{dy}{dx}\right] + \left.\frac{\partial z}{\partial s}\right)_{r,y}\left[\left.\frac{\partial s}{\partial x}\right)_y + \left.\frac{\partial s}{\partial y}\right)_x \frac{dy}{dx}\right] + \left.\frac{\partial z}{\partial y}\right)_{r,s}\frac{dy}{dx}$$

2) a) let $y+2 = m\,x^{3/5}$ $\Longrightarrow$ $\lim\limits_{x\to 0}$ gives an answer $\dfrac{1+m^5}{4-m^5}$

along x axis $\xRightarrow{m\to\infty}$ $\lim = -1$ $\quad$ ∴ limit Does not exist.
along $y=-2$ $\xRightarrow[m=0]{}$ $\lim = 1/4$

b) 2

3) $\dfrac{\partial v}{\partial x} = -2x$ provided $u \neq 0,\ z \neq u^2$; $\dfrac{\partial z}{\partial y} = \dfrac{2u^2y-1}{z-u^2}$ $\quad u \neq 0,\ z \neq u^2$

4) Since Max rate of Chang is $\pm\left|\nabla f|_p\right| = \pm\sqrt{6}$ and $-3 < -\sqrt{6}$.
there is no direction at which the rate of Change is equal to -3.

5)

$x = 1-2t$		$x = 1+2t$		$x = 1+t$
$y = -2+12t$	or	$y = -2-12t$	or	$y = -2-6t$
$z = 1-6t$		$z = 1+6t$		$z = 1+3t$

6) $\nabla(x^2 - xyz + y + 2z - 3)\Big|_{(1,0,1)} = (2,0,2) = \vec{N}$ (vector normal to the Surface)

$(1,0,1) \times (1,1,1) = (-1,0,1) = \vec{V} \leftarrow$ (vector along the Curve at the given point)

$\vec{N}\cdot\vec{V} = (2,02)\cdot(-1,0,1) = 2-2 = 0$

∴ the Curve is tangent to the Surface @ point (1,0,1)

(Fall 07) plankson@yahoo.com

Answers to Dec. 10/2007 Math 2130 Final by Dawit Yohannes

1) $3x+3y+4z-10=0$

2) $\int_0^{\pi/2} \sqrt{8-8\sin t\cos t}\,dt = \int_0^{\pi/2} 2\sqrt{2-\sin 2t}\,dt$

3) $\vec{N}_1=(1,-1,1)$, $\vec{N}_2=(2,1,3)$ $\Rightarrow \cos\gamma = \dfrac{|\vec{N}_1\cdot\vec{N}_2|}{|\vec{N}_1||\vec{N}_2|} = \dfrac{4}{\sqrt{42}} = \dfrac{4\sqrt{42}}{42} = \dfrac{2\sqrt{42}}{21}$

4) $(3r^2+s^4)\left[\dfrac{y}{r}(2z)+\dfrac{z}{r}\right] + (4rs^3)(x\cos xy)(2z)$

5) $\dfrac{4\sqrt{46}}{23}$

6) $b=\pm 5$

7) $-\begin{vmatrix} -1 & 3x^2u^2+v^2 \\ 2yv^5 & 4xu^3+z \end{vmatrix} \Big/ \begin{vmatrix} 2uv-1 & 3x^2u^2+v^2 \\ 5v^4y^2-6 & 4xu^3+z \end{vmatrix}$

Note: $\begin{vmatrix} a & b \\ c & d\end{vmatrix} = -\begin{vmatrix} b & a \\ d & c\end{vmatrix}$
$\begin{vmatrix} a & b \\ c & d\end{vmatrix} = -\begin{vmatrix} d & c \\ b & a\end{vmatrix}$
(from properties of determinants)

8) 2

9) $\dfrac{2\pi}{\sqrt{5}}\int_0^2\int_y^{\sqrt{6-y}} (10-2x-y)\,dx\,dy$

10) $\int_0^{2\pi}\int_0^{\sqrt{3}} \sqrt{1+4r^2}\,r\,dr\,d\theta = 4\int_0^{\pi/2}\int_0^{\sqrt{3}} \sqrt{1+4r^2}\,r\,dr\,d\theta.$

11) $\int_1^3\int_{1-y}^{\sqrt[3]{y-1}} f(y-2)\,dx\,dy$

12) $\int_0^1\int_z^{2-z}\int_0^2 (x^2+y-z^3)\,dx\,dy\,dz$

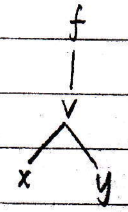
13) $4\int_0^{\pi/2}\int_0^2\int_0^{2r} r\,dz\,dr\,d\theta = \dfrac{32\pi}{3}$

(Winter 08) plankion@yahoo.com

Answers to april 22/2008 Math 2130 Final by Dawit Yohannes

1) $30/\sqrt{90} = \sqrt{10}$

2) $9x+2y-6z-11=0$ or $-9x-2y+6z+11=0$

3) 

let $v = x^3+y^2 \Rightarrow u = f(v)+g(v)$

$$\frac{\partial u}{\partial x} = \frac{du}{dv}\frac{\partial v}{\partial x} = (f'(v)+g'(v))(3x^2)$$

$$\frac{\partial u}{\partial y} = \frac{du}{dv}\frac{\partial v}{\partial y} = (f'(v)+g'(v))(2y)$$

$$2y\left[(3x^2)(f'(v)+g'(v))\right] - 3x^2\left[2y(f'(v)+g'(v))\right] = 0$$

4) $\dfrac{e^x - \cos x}{6u}$

5) $e^{1/4}-1$

6) Max $= \sqrt{2}$, min $=0$

7) a) $\displaystyle\int_1^4\int_{(x-2)^2}^{x} (x^2+y)\,x\,dy\,dx$ b) $\displaystyle\int_1^4\int_{(x-2)^2}^{x} (x^2+y)\left(\frac{4x-3y+1}{5}\right)^2 dy\,dx.$

c) $M = \displaystyle\int_1^4\int_{(x-2)^2}^{x} (x^2+y)\,dy\,dx$; $\bar{x} = \dfrac{1}{M}\displaystyle\int_1^4\int_{(x-2)^2}^{x} x(x^2+y)\,dy\,dx$; $\bar{y} = \dfrac{1}{M}\displaystyle\int_1^4\int_{(x-2)^2}^{x} y(x^2+y)\,dy\,dx$

8) a) $\displaystyle\iint_R \sqrt{1+4(x^2+y^2)}\,dA$ b) Surface area of $z=y^2-x^2$ bounded by $x^2+y^2=1$ and $x^2+y^2=4$.

c) $\dfrac{\pi}{6}\left(17\sqrt{17}-5\sqrt{5}\right)$

9) $\displaystyle\int_0^3\int_{z/3}^{4-z}\int_0^3 dy\,dx\,dz$

10) a) $(R,\phi,\theta) = (4, \pi/6, \pi/4)$

b) $(r,\theta,z) = (\sqrt{2}, \pi/4, \sqrt{6})$

11) $\pi/2$

12) $\dfrac{135k\pi}{2}.$

plankion@yahoo.Com.

(Fall 08)

Answers to Dec 11/2008 Math 2130 Final by Dawit yohannes

1.) the question has no solution because the two lines don't intersect. (They are skew-lines)

2) $\frac{2e^t}{3x} + \frac{6ty^2}{e^y}$

3) Max $= 3/2$; Min $= -3$

4) $1808g$

5) $8\sqrt{2}(\sqrt{8}-\sqrt{6})\pi$

6) $\int_0^1\int_0^{\sqrt{1-z^2}}\int_0^y f(x,y,z)\,dx\,dy\,dz$; $\int_0^1\int_0^{\sqrt{1-y^2}}\int_0^y f(x,y,z)\,dx\,dy\,dz$

$\int_0^1\int_0^{\sqrt{1-x^2}}\int_x^{\sqrt{1-z^2}} f(x,y,z)\,dy\,dz\,dx$; $\int_0^1\int_0^{\sqrt{1-z^2}}\int_x^{\sqrt{1-z^2}} f(x,y,z)\,dy\,dx\,dz$

$\int_0^1\int_0^y\int_x^{\sqrt{1-y^2}} f(x,y,z)\,dz\,dx\,dy$; $\int_0^1\int_x^1\int_0^{\sqrt{1-y^2}} f(x,y,z)\,dz\,dy\,dx$

7) $\int_0^1\int_{x^3}^{\sqrt{2-x}} \rho\left(\frac{3x-2y-6}{\sqrt{13}}\right)^2 dy\,dx$ 8) $\frac{3\pi-2}{6}$

9) a) $R = \frac{1}{\sqrt{\cos 2\phi}} = \sqrt{\sec 2\phi}$ b) $z = \frac{\sqrt{x^2+y^2}-x^2-y-1}{2(x^2+y^2)}$

10) $\int_0^{2\pi}\int_0^{\pi/3}\int_{\sec\phi}^{2} R^2 \sin\phi\, dR\,d\phi\,d\theta = 4\int_0^{\pi/2}\int_0^{\pi/3}\int_{\sec\phi}^{2} R^2\sin\phi\,dR\,d\phi\,d\theta$

(Winter 09)

Answers to April 21/2009 Math 2130 Final

by Dawit Yohannes (plankion@yahoo.com)

1) Given any $\epsilon > 0$, We can find $\delta > 0$ Such that $|f(x,y) - L| < \epsilon$ whenever $0 < \left|\sqrt{(x-a)^2-(y-b)^2}\right| < \delta$.
provided (x,y) is in the domain of $f(x,y)$

2) Show that $\frac{\partial^2 f}{\partial x^2} = -36 \sin 6x \sin 2y$ and $\frac{\partial^2 f}{\partial y^2} = -4 \sin 6x \sin 2y$

3) $6x - 3y - 2z + 2 = 0$

4) $\sin 4$

5) $$-(6uv^3 + ve^{uv}) \frac{\begin{vmatrix} 24u^2x^3 & 5y^3 \\ 56x^6y & -15u^2v^2 \end{vmatrix}}{\begin{vmatrix} 12ux^4 & 5y^3 \\ -10uv^3 & -15u^2v^2 \end{vmatrix}} - (9u^2v^2 + ue^{uv}) \frac{\begin{vmatrix} 12ux^4 & 24u^2x^3 \\ -10uv^3 & 56x^6y \end{vmatrix}}{\begin{vmatrix} 12ux^4 & 5y^3 \\ -10uv^3 & -15u^2v^2 \end{vmatrix}}$$

6) a) $x = t,\ y = t^2 - t + 3,\ z = 3t^2 + t - 3$ b) $\int_1^3 \sqrt{3 + 8t + 40t^2}\, dt$

7) $(0,0)$ yields a saddle point ; $(2,2)$ yields a relative minimum.

8) Maximum Value $= 4/3$, Minimum Value $= -4/3$

9) $\int_0^{2\pi}\int_1^3 kr^2(r\cos\theta + 3)^2\, dr\, d\theta$ (about a vertical tangent)

10) $\int_0^4 \int_{(y-2)^2}^{12-2(y-2)^2} \sqrt{1 + 24xy + 13(x^2+y^2)}\, dx\, dy$

11) $M = \int_{-2}^{3}\int_0^{6+x-x^2}\int_0^{20-2(x+y)} kz\, dz\, dy\, dx$; $\bar{x} = \frac{1}{M}\int_{-2}^{3}\int_0^{6+x-x^2}\int_0^{20-2(x+y)} kxz\, dz\, dy\, dx$; $\bar{y} = \frac{1}{M}\int_{-2}^{3}\int_0^{6+x-x^2}\int_0^{20-2(x+y)} kyz\, dz\, dy\, dx$

$\bar{z} = \frac{1}{M}\int_{-2}^{3}\int_0^{6+x-x^2}\int_0^{20-2(x+y)} kz^2\, dz\, dy\, dx$.

12) $\int_0^1\int_{2x}^{2}\int_0^{36-9y^2} dz\, dy\, dx$, $\int_0^2\int_0^{y/2}\int_0^{36-9y^2} dz\, dx\, dy$,

$\int_0^{36}\int_0^{\frac{1}{3}\sqrt{36-z}}\int_0^{y/2} dx\, dy\, dz$, $\int_0^2\int_0^{36-9y^2}\int_0^{y/2} dx\, dz\, dy$,

$\int_0^1\int_0^{36(1-x^2)}\int_{2x}^{\frac{1}{3}\sqrt{36-z}} dy\, dz\, dx$, $\int_0^{36}\int_0^{\frac{1}{6}\sqrt{36-z}}\int_0^{\frac{1}{3}\sqrt{36-z}} dy\, dx\, dz$

13) $\frac{32(2-\sqrt{2})\pi}{5}$

(Fall 09)

plankeon@yahoo.com

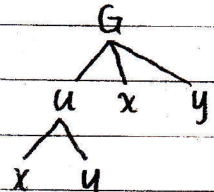
Answers to Dec. 18/2009 Math 2130 Final

by Dawit Yohannes

1) a) $y = x^2/4$ b) $x = -t,\ y = t^2/4,\ z = 5 - t^2/4$ c) $\int_{-\sqrt{20}}^{0} \sqrt{1 + t^2/2}\, dt$

2.) $9/\sqrt{117} = \dfrac{3\sqrt{13}}{13}$ 3) $\dfrac{1}{\sqrt{266}}(\ln 2 - 16)$

4.) let $u = 3x - 2y^2 \implies G(x,y) = f(u) + xy$, with $u = u(x,y)$



$$\frac{\partial G}{\partial x} = \left.\frac{\partial G}{\partial u}\right)_{x,y} \frac{\partial u}{\partial x} + \left.\frac{\partial G}{\partial x}\right)_{u,y} = f'(u)\cdot 3 + y$$

$$\frac{\partial G}{\partial y} = \left.\frac{\partial G}{\partial u}\right)_{x,y} \frac{\partial u}{\partial y} + \left.\frac{\partial G}{\partial y}\right)_{x,u} = f'(u)(-4y) + x$$

$$LHS = 4y\left[\cancel{f'(u)\cdot 3} + y\right] + 3\left[\cancel{f'(u)\cdot(-4y)} + x\right] = 4y^2 + 3x = RHS$$

5.) (0,0) yields rel-min; (1,1) yields Saddle point

6) (0,0) yields a Saddle point

7) max-value $= 1/27$ 8) $1/\pi$

9.) $\dfrac{\pi}{6}\left[(1+4a^2)^{3/2} - 1\right]$

10) a) $\int_0^2 \int_{x^2-2}^{\frac{x+2}{2}} 2\pi(x+4)\, dy\, dx$ b) $\int_0^2 \int_{x^2-2}^{\frac{x+2}{2}} (x^2+y^2)\left(\dfrac{x-2y+2}{\sqrt{5}}\right)^2 dy\, dx$

11) a) $4\int_0^{\pi/2}\int_0^{\sqrt{2}}\int_r^{\sqrt{4-r^2}} r\, dz\, dr\, d\theta$ b) $4\int_0^{\pi/2}\int_0^{\pi/4}\int_0^{2} R^2 \sin\phi\, dR\, d\phi\, d\theta.$