UNIVERSITY OF MANITOBA

DATE: February 7, 2013 MIDTERM I

TITLE PAGE

COURSE: MATH 2130 TIME: 70 minutes

EXAMINATION: Engineering Mathematical Analysis 1 EXAMINER: M. Davidson

FAMILY NAME: (Print in ink) Solutions

GIVEN NAME(S): (Print in ink) ________

STUDENT NUMBER: ________

SIGNATURE: (in ink) ________

(I understand that cheating is a serious offense)

INSTRUCTIONS TO STUDENTS:

This is a 70 minute exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, and 5 pages of questions. Please check that you have all the pages.

The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 40 points.

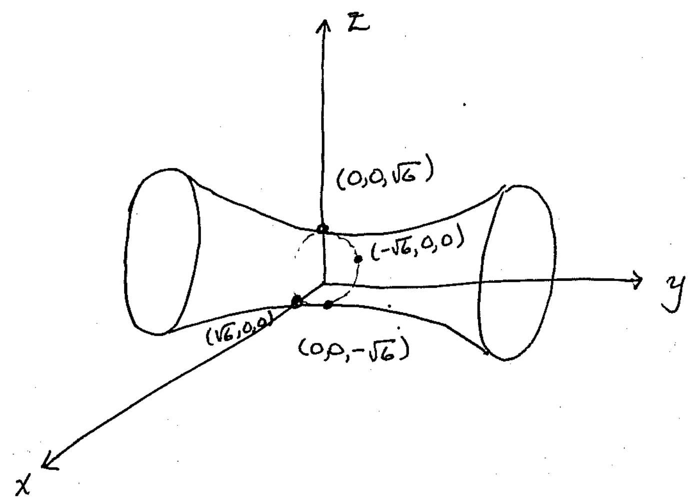
Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	7	
2	8	
3	8	
4	6	
5	5	
6	6	
Total:	40	

[4] 1. (a) Identify and sketch the surface $z^2 = 6 - x^2 + y^2$.

$x^2 - y^2 + z^2 = 6$

is a hyperboloid of one sheet.



[3] (b) Find the projection of $z^2 = 6 - x^2 + y^2$, $z + y = 3$ onto the xy-plane.

From $z + y = 3$ we get
$z = 3 - y$

Substituting into $z^2 = 6 - x^2 + y^2$, we find

$$(3-y)^2 = 6 - x^2 + y^2$$
$$9 - 6y + y^2 = 6 - x^2 + y^2$$
$$9 - 6y = 6 - x^2$$
$$-6y = -3 - x^2$$
$$y = \frac{3 + x^2}{6}$$

[8] 2. Find the distance between the line

$$x = 5 + t,\ y = -1 - t,\ z = 8 + 3t$$

and the line

$$\frac{x-4}{-2} = \frac{y-1}{2} = \frac{z-2}{-6}.$$

From the first line, we find a vector in the direction of the line, $\vec{V_1} = \langle 1, -1, 3\rangle$ and a point on the line $P_1 = (5, -1, 8)$.

From the second line, we find a vector in the direction of the line, $\vec{V_2} = \langle -2, 2, -6\rangle$ and a point on the line $P_2 = (4, 1, 2)$.

Since $\vec{V_2} = -2\vec{V_1}$, the lines are parallel.
So the distance between the two lines is the distance between the point $(5, -1, 8)$ and the second line.

Method from text

$\overrightarrow{P_1P_2} = \langle -1, 2, -6\rangle$

$\vec{V} = \langle 1, -1, 3\rangle$

$$\overrightarrow{P_1P_2} \times \vec{V} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 2 & -6 \\ 1 & -1 & 3 \end{vmatrix} = \langle 0, -3, -1\rangle$$

$$(\overrightarrow{P_1P_2} \times \vec{V}) \times \vec{V} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 0 & -3 & -1 \\ 1 & -1 & 3 \end{vmatrix} = \langle -10, -1, 3\rangle$$

$$\| \langle -10, -1, 3\rangle \| = \sqrt{100+1+9} = \sqrt{110}$$

So $d = \left| \langle -1, 2, -6\rangle \cdot \frac{1}{\sqrt{110}} \langle -10, -1, 3\rangle \right|$

$$= \frac{|10-2-18|}{\sqrt{110}} = \frac{10}{\sqrt{110}}$$

Method from exercises

$\overrightarrow{P_1P_2} = \langle -1, 2, -6\rangle$

$\vec{V} = \langle 1, -1, 3\rangle$

$\|\vec{V}\| = \sqrt{1+1+9} = \sqrt{11}$

$$\hat{V} = \frac{1}{\sqrt{11}} \langle 1, -1, 3\rangle$$

$$\overrightarrow{P_1P_2} \times \hat{V} = \frac{1}{\sqrt{11}} \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 2 & -6 \\ 1 & -1 & 3 \end{vmatrix}$$

$$= \frac{1}{\sqrt{11}} \langle 0, -3, -1\rangle$$

So $d = \|\overrightarrow{P_1P_2} \times \hat{V}\|$

$$= \left\| \frac{1}{\sqrt{11}} \langle 0, -3, -1\rangle \right\|$$

$$= \frac{\sqrt{10}}{\sqrt{11}}$$

[8] 3. Find a vector representation of the curve of intersection of $z = x^2 + 2y^2$ and $4x + 4y + z = 10$ oriented so it is counterclockwise when viewed from far out on the positive z axis.

From $4x+4y+z=10$ we find $z = 10-4x-4y$ and we substitute into $z = x^2+2y^2$ to find

$$x^2+2y^2 = 10-4x-4y$$
$$x^2+4x+2y^2+4y = 10$$
$$(x^2+4x+4)+2(y^2+2y+1) = 10+4+2$$
$$(x+2)^2+2(y+1)^2 = 16$$
$$\frac{(x+2)^2}{16} + \frac{2(y+1)^2}{16} = 1$$
$$\left(\frac{x+2}{4}\right)^2 + \left(\frac{y+1}{2\sqrt{2}}\right)^2 = 1$$

We now let $\frac{x+2}{4} = \cos t$ and $\frac{y+1}{2\sqrt{2}} = \sin t$ $(0 \le t \le 2\pi)$

[Note that counterclockwise from z^+ is the "usual" direction, so we leave $\sin t$ positive]

So $x = 4\cos t - 2$ $y = 2\sqrt{2}\sin t - 1$

$$z = 10 - 4(4\cos t - 2) - 4(2\sqrt{2}\sin t - 1)$$
$$= 10 - 16\cos t + 8 - 8\sqrt{2}\sin t + 4$$
$$= 22 - 16\cos t - 8\sqrt{2}\sin t$$

So a vector representation of the curve is

$$\vec{r}(t) = \left\langle 4\cos t - 2,\ 2\sqrt{2}\sin t - 1,\ 22 - 16\cos t - 8\sqrt{2}\sin t \right\rangle$$

for $0 \le t \le 2\pi$.

[6] 4. Find a unit tangent vector to the curve $3x + y = 7$, $z + x^2 + 2y = 9$ at the point $(2, 1, 3)$.

To parametrize the curve, we let $x = t$, then $y = 7 - 3t$ and $z = 9 - t^2 - 2(7-3t) = 9 - t^2 - 14 + 6t = -5 + 6t - t^2$.

So $\vec{r}(t) = \langle t, 7-3t, -5+6t-t^2 \rangle$

$\vec{r}\,'(t) = \vec{T}(t) = \langle 1, -3, 6-2t \rangle$

The point $(2,1,3)$ is at $t = 2$, so

$\vec{T}(2) = \langle 1, -3, 2 \rangle$

$\|\vec{T}(2)\| = \sqrt{1+9+4} = \sqrt{14}$

So $\hat{T} = \frac{1}{\sqrt{14}} \langle 1, -3, 2 \rangle$ is a unit tangent vector.

[5] 5. Evaluate the following limit, or show that it does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{2x^2y^8}{x^4 + y^16}.$$

If we approach along $x = 0$, we find

$$\lim_{(0,y)\to(0,0)} \frac{0}{y^{16}} = 0$$

If we approach along $x = y^4$, we find

$$\lim_{(y^4,y)\to(0,0)} \frac{2(y^4)^2y^8}{(y^4)^4 + y^{16}} = \lim_{y\to 0} \frac{2y^{16}}{2y^{16}} = 1$$

Since different approaches give different values, the limit does not exist.

(alternately)

Approach along $x = my^4$, we find

$$\lim_{(my^4,y)\to(0,0)} \frac{2(my^4)^2y^8}{(my^4)^4 + y^{16}} = \lim_{y\to 0} \frac{2m^2y^{16}}{m^4y^{16}+y^{16}} = \frac{2m^2}{m^4+1}$$

Since the limit is dependant upon m, the limit does not exist.

[6] 6. Set up but *do not evaluate* an integral for the length of the curve

$$\vec{r}(t) = \langle 2\sin(3t), 2\cos(3t), \tan(t)\rangle$$

from the point $(0, 2, 0)$ to the point $(\sqrt{2}, -\sqrt{2}, 1)$. Simplify the integrand.

The point $(0,2,0)$ is found when

$2\sin 3t = 0$

$2\cos 3t = 2$

$\tan t = 0$

from $2\sin 3t = 0$ we get $3t = 0$ or $3t = \pi$

since $\cos(0) = 1$ and $\cos\pi = -1$ we see that $t = 0$.

The point $(\sqrt{2}, -\sqrt{2}, 1)$ is found when

$2\sin 3t = \sqrt{2}$

$2\cos 3t = -\sqrt{2}$

$\tan t = 1$

from $2\sin 3t = \sqrt{2}$

$\sin 3t = \frac{\sqrt{2}}{2}$ so

$3t = \frac{\pi}{4}$ or $3t = \frac{3\pi}{4}$

So $t = \frac{\pi}{12}$ or $t = \frac{\pi}{4}$

Since $\tan(\pi/12) \neq 1$ but $\tan(\frac{\pi}{4}) = 1$ we see $t = \pi/4$

From the formula

$$\text{Length} = \int_\alpha^\beta \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$$

We get

$$\int_0^{\pi/4} \sqrt{(6\cos(3t))^2 + (-6\sin(3t))^2 + (\sec^2 t)^2}\, dt$$

$$= \int_0^{\pi/4} \sqrt{36\cos^2(3t) + 36\sin^2(3t) + \sec^4 t}\, dt$$

$$= \int_0^{\pi/4} \sqrt{36 + \sec^4 t}\, dt.$$