UNIVERSITY OF MANITOBA

DATE: March 14, 2013 MIDTERM II

TITLE PAGE

COURSE: MATH 2130 TIME: 70 minutes

EXAMINATION: Engineering Mathematical Analysis 1 EXAMINER: M. Davidson

FAMILY NAME: (Print in ink) Solutions

GIVEN NAME(S): (Print in ink)

STUDENT NUMBER:

SIGNATURE: (in ink)

(I understand that cheating is a serious offense)

INSTRUCTIONS TO STUDENTS:

This is a 70 minute exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page, and 4 pages of questions. Please check that you have all the pages.

The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 40 points.

Answer all questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	5	
2	5	
3	7	
4	4	
5	8	
6	11	
Total:	40	

[5] 1. Find an equation in simplified form, of the plane ~~normal~~ tangent to the surface $z^2 - x^2 z + xy^2 = 3y + 10$ at the point $(3, 1, -1)$.

Let $F(x,y,z) = z^2 - x^2 z + xy^2 - 3y - 10$

Then $\nabla F = \langle -2xz + y^2, 2xy - 3, 2z - x^2 \rangle$

So $\nabla F \big|_{(3,1,-1)} = \langle -2(3)(-1) + (1)^2, 2(3)(1) - 3, 2(-1) - (3)^2 \rangle$

$= \langle 7, 3, -11 \rangle$

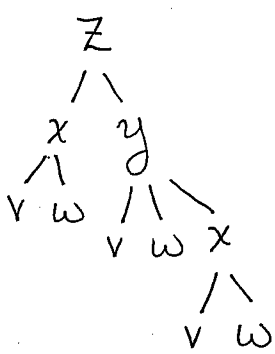
So the tangent plane is

$7(x-3) + 3(y-1) - 11(z+1) = 0$

In simplified form:

$7x + 3y - 11z = 35$

[5] 2. If $z = f(x,y)$, $y = g(v,w,x)$ and $x = h(v,w)$, find the chain rule for $\left.\dfrac{\partial z}{\partial v}\right)_w$.



$$\left.\frac{\partial z}{\partial v}\right)_w = \left.\frac{\partial z}{\partial x}\right)_y \cdot \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \cdot \left.\frac{\partial y}{\partial v}\right)_{w x} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial x} \cdot \frac{\partial x}{\partial v}$$

[7] 3. Find the rate of change of $f(x, y, z) = 3yz^2 - x^2y^3$ at the point $(-1, 1, 2)$ in the direction $\vec{r}(t) = \langle 3t + 2, t^2, t^2 + t + 2\rangle$.

$$\nabla f = \langle -2xy^3, 3z^2 - 3x^2y^2, 6yz \rangle$$

$$\nabla f \Big|_{(-1,1,2)} = \langle -2(-1)(1)^3, 3(2)^2 - 3(-1)^2(1)^2, 6(1)(2) \rangle$$

$$= \langle 2, 9, 12 \rangle$$

$$\frac{d\vec{r}}{dt} = \vec{T}(t) = \langle 3, 2t, 2t+1 \rangle \qquad \left(\begin{array}{r} 3t+2=-1 \\ 3t=-3 \\ t=-1 \end{array} \right)$$

$$\vec{T}(-1) = \langle 3, -2, -1 \rangle$$

$$\hat{T}(-1) = \frac{1}{\sqrt{14}} \langle 3, -2, -1 \rangle$$

So $D_{\vec{v}} f = \langle 2, 9, 12 \rangle \cdot \left(\frac{1}{\sqrt{14}} \langle 3, -2, -1 \rangle \right)$

$$= \frac{6 - 18 - 12}{\sqrt{14}} = \frac{-24}{\sqrt{14}} \left(= \frac{-24\sqrt{14}}{14} = \frac{-12\sqrt{14}}{7} \right)$$

[4] 4. Find all directions where the rate of change of $f(x, y) = x^2 + x^3y^2 + 7y$ is equal to 0 at the point $(2, -1)$. (Any answer should be in the form of a vector.)

$$\nabla f = \langle 2x + 3x^2y^2, 2x^3y + 7 \rangle$$

$$\nabla f_{(2,-1)} = \langle 4 + 12, -16 + 7 \rangle = \langle 16, -9 \rangle$$

If $\vec{v} = \langle a, b \rangle$ then $D_{\vec{v}} f = 0$ when

$$\langle 16, -9 \rangle \cdot \langle a, b \rangle = 0$$

$$16a - 9b = 0$$

$$a = \frac{9}{16} b$$

So the vectors are $\langle \frac{9}{16} b, b \rangle$ $\left\{ \text{or } \langle a, \frac{16}{9} a \rangle \text{ or } \langle 9a, 16a \rangle \right\}$

[Alternately, there are two directions in which the rate of change is 0, given by the unit vectors

$\vec{v}_1 = \frac{1}{\sqrt{337}} \langle 9, 16 \rangle$ and $\vec{v}_2 = \frac{1}{\sqrt{337}} \langle -9, -16 \rangle$]

[8] 5. The following equations

$$u^3y^2 + 2xz^2 + v = u + y\,,$$
$$ux^2 + v^3y^2 + zy = z^2 - v$$

define x and z as functions of the other variables. Find $\dfrac{\partial z}{\partial v}$.

Let $F(u, v, x, y, z) = u^3y^2 + 2xz^2 + v - u - y$

$G(u, v, x, y, z) = ux^2 + v^3y^2 + zy - z^2 + v$

$$\frac{\partial z}{\partial x} = -\frac{\dfrac{\partial(F,G)}{\partial(x,v)}}{\dfrac{\partial(F,G)}{\partial(x,z)}} = \frac{-\begin{vmatrix} F_x & F_v \\ G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_x & F_z \\ G_x & G_z \end{vmatrix}}$$

$$= \frac{-\begin{vmatrix} 2z^2 & 1 \\ 2xu & 3v^2y^2+1 \end{vmatrix}}{\begin{vmatrix} 2z^2 & 4xz \\ 2xu & y-2z \end{vmatrix}}$$

$$= \frac{-\left(2z^2(3v^2y^2+1) - 2xu\right)}{\left(2z^2(y-2z) - 8x^2uz\right)}$$

$$\left(= \frac{-6v^2y^2z^2 - 2z^2 + 2xu}{2yz^2 - 4z^3 - 8x^2uz} = \frac{-3v^2y^2z^2 - z^2 + xu}{yz^2 - 2z^3 - 4x^2uz} \right)$$

[11] 6. Find and classify all critical points of

$$f(x,y) = 2x^2y + xy^2 - 6xy\,.$$

$$f_x = 4xy + y^2 - 6y = y(4x + y - 6)$$
$$f_y = 2x^2 + 2xy - 6x = x(2x + 2y - 6)$$

$f_x = 0$ when $y = 0$ or $4x + y = 6$

$f_y = 0$ when $x = 0$ or $2x + 2y = 6$

$\begin{pmatrix} y=0 \\ x=0 \end{pmatrix}$ $\begin{pmatrix} 4x+y=6 \\ x=0 \end{pmatrix}$ $\begin{pmatrix} y=0 \\ 2x+2y=6 \end{pmatrix}$ $\left(\begin{matrix} 4x+y=6 \\ 2x+2y=6 \end{matrix} \Rightarrow \begin{matrix} 4x+y=6 \\ 4x+4y=12 \\ \hline -3y=-6 \\ y=2 \\ \Rightarrow x=1 \end{matrix}\right)$

$(0,0)$ $(0,6)$ $(3,0)$ $(1,2)$

$f_{xx} = 4y$ (A) $f_{xy} = 4x + 2y - 6$ (B) $f_{yy} = 2x$ (C)

Critical point	A (4y)	B (4x+2y−6)	C (2x)	$B^2 - AC$	Classification
(0,0)	0	−6	0	36	saddle point
(0,6)	24	6	0	36	saddle point
(3,0)	0	6	6	36	saddle point
(1,2)	8	2	2	−12	relative minimum (Since A > 0)