MATH 2500 Assignment #2

Due: February 15, 2013, Before Class (12:30)

Reminder: all assignments *must* be accompanied by a signed copy of the honesty declaration available on my website.
Assignments are to be handed in on $8\frac{1}{2} \times 11$ paper, single sided, no ragged edges, stapled in the top left hand corner with the honesty declaration as the first page.

1. For each of the following linear Diophantine equations, find all integer solutions for x and y:
   (a) $1003x + 3164y = 52$
   (b) $3633x + 1540y = 52$
   (c) $4381x + 1352y = 52$

2. For each of the following linear modular congruences, find all solutions:
   (a) $16416x \equiv 328 \pmod{36012}$
   (b) $1158x \equiv 732 \pmod{3660}$
   (c) $113x \equiv 107 \pmod{248}$

3. For each of the following systems of linear modular congruences, if possible, write as a single congruence:
   (If it is not possible, explain why it is not possible.)
   (a) $x \equiv 2 \pmod{5}$
   $x \equiv 13 \pmod{22}$
   $x \equiv 20 \pmod{49}$
   (b) $x \equiv 22 \pmod{153}$
   $x \equiv 41 \pmod{119}$
   (c) $x \equiv 17 \pmod{39}$
   $x \equiv 43 \pmod{91}$

4. If $1743 \equiv 2406 \pmod{m}$ is a true statement, then what are the possible values of m?

5. (a) Prove that if $d|m$ and $a \equiv b \pmod{m}$, then $a \equiv b \pmod{d}$.
   (b) Twin primes are pairs of primes that differ by two. For example, 5 and 7 are twin primes, 11 and 13 are twin primes.
   Prove that if p and q are twin primes other than the pair 3 and 5, then $pq \equiv -1 \pmod{6}$.
   (Hint: see Section 4, Problem 9, page 32.)