MATH 2500 Assignment #3

Due: March 4, 2012, Before Class (12:30)

Reminder: all assignments *must* be accompanied by a signed copy of the honesty declaration available on my website.
Assignments are to be handed in on $8\frac{1}{2} \times 11$ paper, single sided, no ragged edges, stapled in the top left hand corner with the honesty declaration as the first page.

1. Find the least residue of the given expression in the given modulus. Name any theorem you may use.
   (a) $1152 \cdot 1151 \cdot 1150 \cdots 109 \cdot 108 \cdot 105 \cdot 104 \cdots 3 \cdot 2 \cdot 1 \pmod{1153}$
   (Note: This is all factors of 1152! except 107 and 106.)
   (b) $13^{56162} \pmod{1147}$
   (c) $1072 \cdot 1071 \cdot 1070 \cdots 533 \cdot 532 \cdot 530 \cdot 529 \cdots 3 \cdot 2 \cdot 1 \pmod{1073}$
   (Note: This is all factors of 1072! except 531.)
   (d) $6^{49144} \pmod{1171}$
   (e) $5^{50008} \pmod{1283}$
   (f) $1068 \cdot 1067 \cdot 1066 \cdots 7 \cdot 6 \cdot 5 \pmod{1069}$
   (Note: This is all factors of 1068! except 4, 3, 2 and 1.)
   (g) $7^{61603} \pmod{1121}$.
2. For each of the following, find $d(n)$, $\sigma(n)$ and $\phi(n)$:
   (a) 205821
   (b) 29766
   (c) 3577392.
3. Decide if:
   (a) 33550336 is perfect.
   (b) 523776 is k-perfect (if so,find the value of k).
   (c) 5020 is one of an amicable pair, and if so find the other number in the pair.
   (d) The aliquot sequence of the number 12496 repeats with a period of 5.
4. Suppose n is an 4-perfect number and $(n, 40) = 1$. Is $40n$ k-perfect number? If so, for what k?
5. Show that, for $n \geq 2$, the sum of all numbers less than n and relatively prime to n is equal to $\dfrac{\phi(n) \cdot n}{2}$.