MATH 2500 Assignment #4

Due: April 3, 2013, Before Class (12:30)

Reminder: all assignments *must* be accompanied by an honesty declaration available on my website.

1. (a) How many primitive roots does 89 have?
   (b) What are the possible orders of an element modulo 89?
   (c) Show that 3 is a primitive root of 89.
   (d) Find 4 other primitive roots. (Note, these should be in least residue.)
   (e) Given that $3^{32} \equiv 4 \pmod{89}$, what is the order of 4 (mod 89)? Of 12 (mod 89)? Of 36 (mod 89)?

2. Which of the following quadratic congruences have solutions? (You do NOT need to find solutions if they exist.)
   (a) $x^2 + 11\,x + 6 \equiv 0 \pmod{53}$
   (b) $7\,x^2 + 3\,x + 30 \equiv 0 \pmod{71}$
   (c) $6\,x^2 + 7\,x + 1 \equiv 0 \pmod{37}$

3. Solve the following Legendre symbols:
   (a) $\left(\frac{3879}{6211}\right)$
   (b) $\left(\frac{2184}{8999}\right)$
   (c) $\left(\frac{3433}{7001}\right)$

4. Which of $2, 3, 4, 5$ are primitive roots of 3623?
   [Hint: Consider Euler's Criterion.]

5. (a) If a has order t modulo m, what is the order of a^2 modulo m? (Hint: separate into the two cases of t odd and t even.) Justify your answer.
   (b) If a has order 3 modulo m and b has order 5 modulo m, what is the order of ab modulo m? Justify your answer.