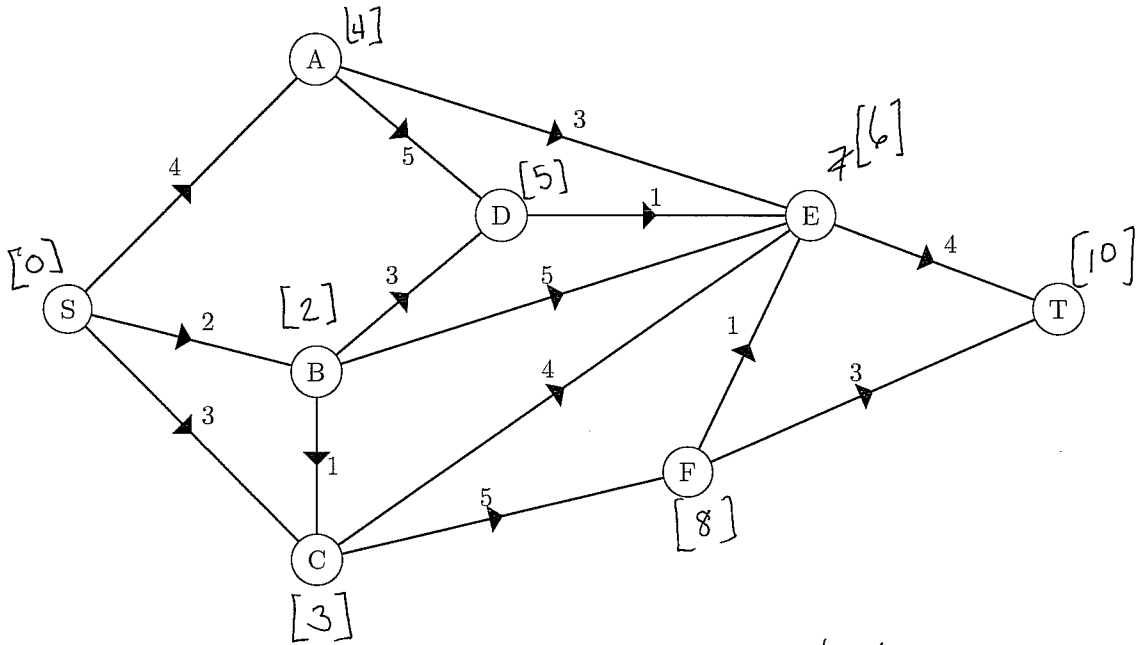
Shortest path, longest path, and Chinese postman problem examples

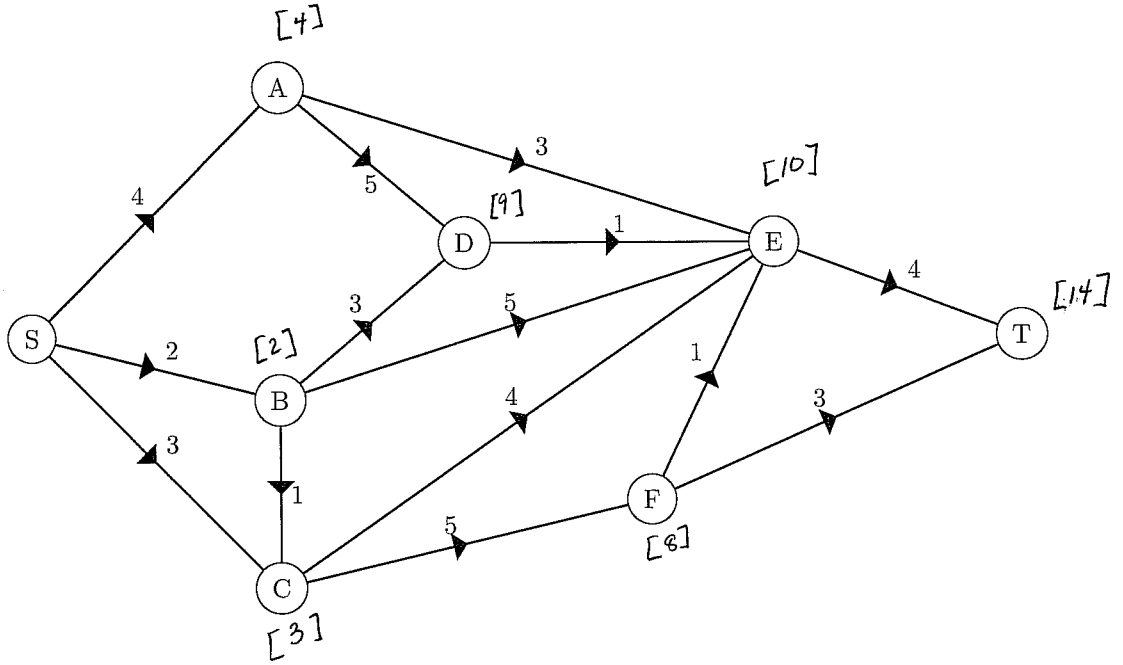
In the following weighted digraph, find the shortest path from node S to node T.



The shortest path has length 10.

$S \to B \to D \to E \to T$

In the following weighted digraph, find the longest path from node S to node T, and then complete the scheduling problem.



S	A	B	C	D	E	F	T
–	S	S	BS	AB	ABCDF	C	EF
					7 7 7 10 9	8	14 11
0	4	2	3	9	10		14

The longest path has length 14

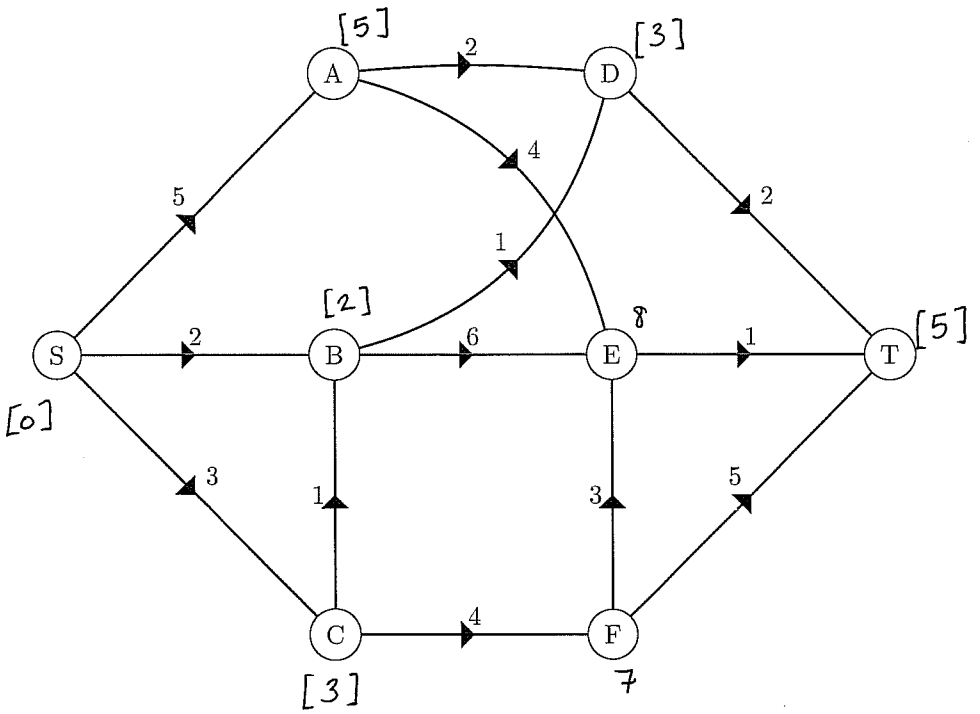
$S \to A \to D \to E \to T$

Complete the following table according to scheduling the events represented in the above graph:

E-Earliest start time L-Latest start time F-Float time

	SA	SB	SC	AD	AE	BD	BE	~~CB~~ BC	CE	CF	DE	ET	FE	FT
E	0	0	0	4	4	2	2	3	3	3	9	10	8	8
L	0	1	1	4	7	6	5	3	6	4	9	10	9	11
F	0	1	1	0	3	4	3	1	3	1	0	0	1	3

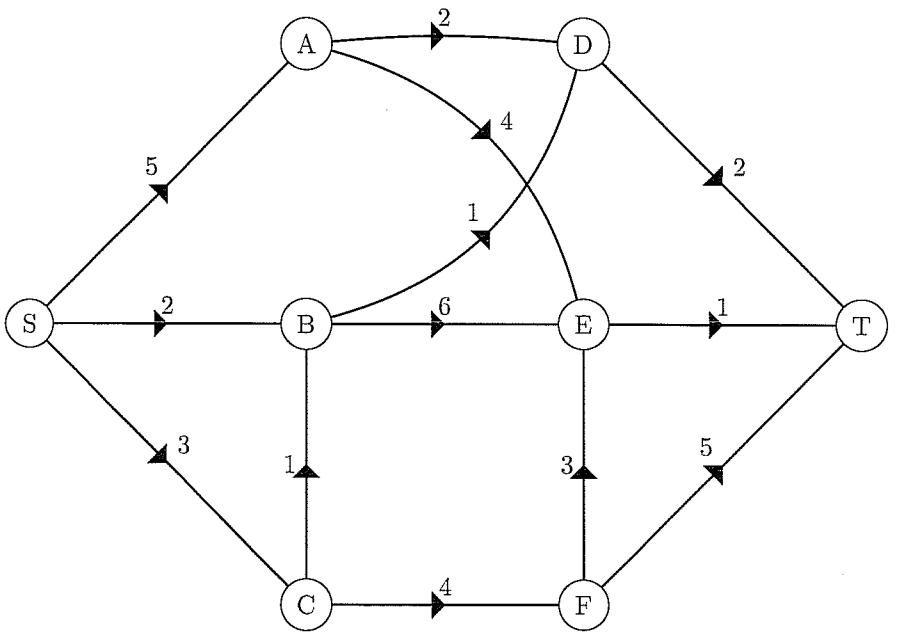
In the following weighted digraph, find the shortest path from node S to node T.



The shortest path from S to T is of weight 5

S → B → D → T

In the following weighted digraph, find the longest path from node S to node T.



S	A	B	C	D	E	F	T
–	S	SC 2 4	S	AB 7 5	ABF 9 10 10	C 7	DEF 9 11 12
[0]	[5]	[4]	[3]	[7]	[10]	[7]	[12]

The longest path is of weight 12 :

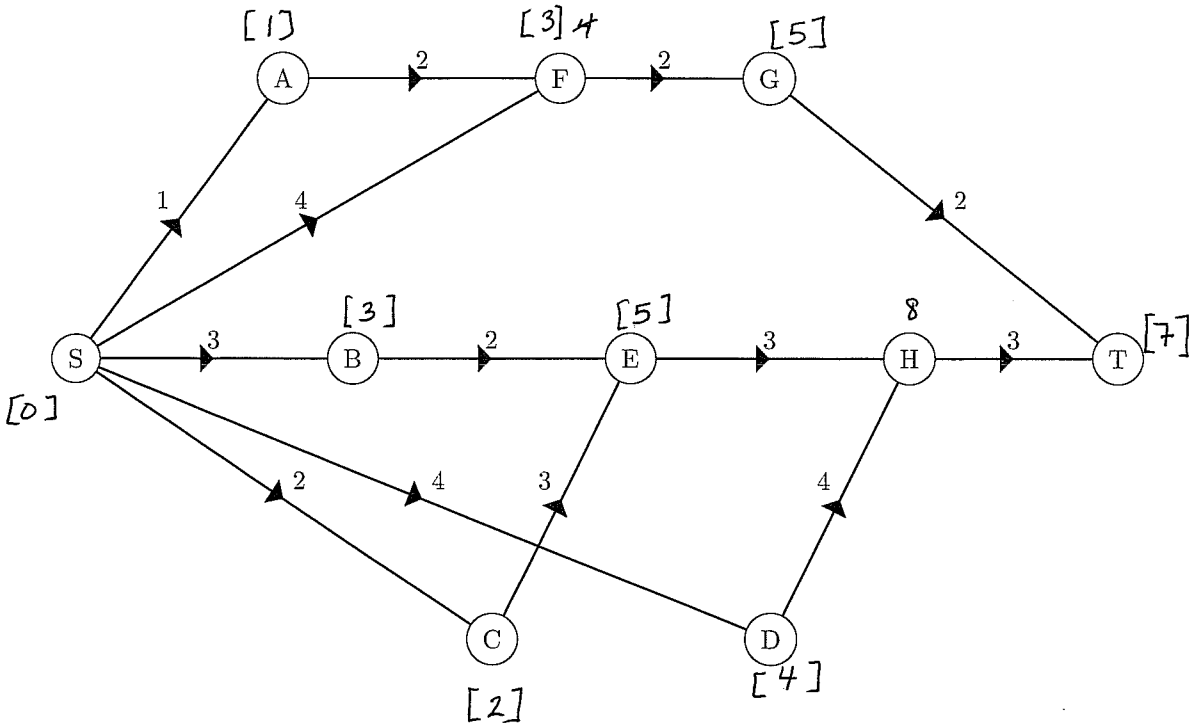
$$S \to C \to F \to T$$

Complete the following table according to scheduling the events represented in the above graph:

E-Earliest start time L-Latest start time F-Float time

	SA	SB	SC	AD	AE	BD	BE	CB	CF	DT	ET	FE	FT
E	0	0	0	5	5	4	4	3	3	7	10	7	7
L	2	3	0	8	7	9	5	4	3	10	11	8	7
F	2	3	0	3	2	5	1	1	0	3	1	1	0

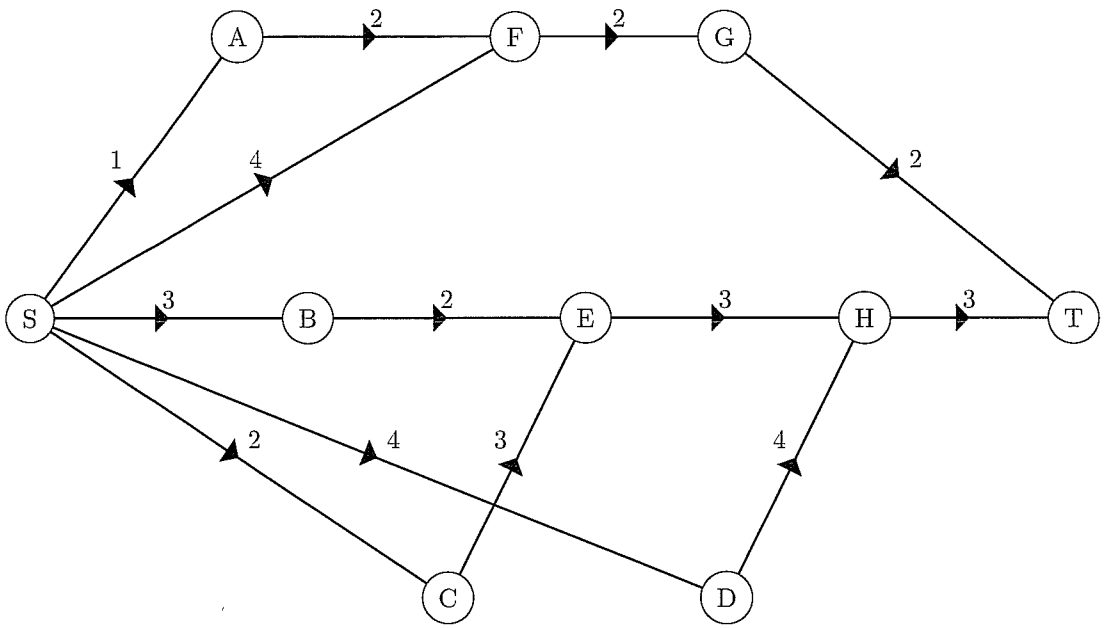
In the following weighted digraph, find the shortest path from node S to node T.



The shortest path from S to T is of weight 7

$S \to A \to F \to G \to T$

In the following weighted digraph, find the longest path from node S to node T.



S	A	B	C	D	E	F	G	H	T
-	S	S	S	S	BC	AS	F	DE	GH
					5 5	3 4	6	8 8	8 11
[0]	[1]	[3]	[2]	[4]	[5]	[4]	[6]	[8]	[11]

The longest path has weight 11:

$S \to B \to E \to H \to T$

$S \to C \to E \to H \to T$

$S \to D \to H \to T$

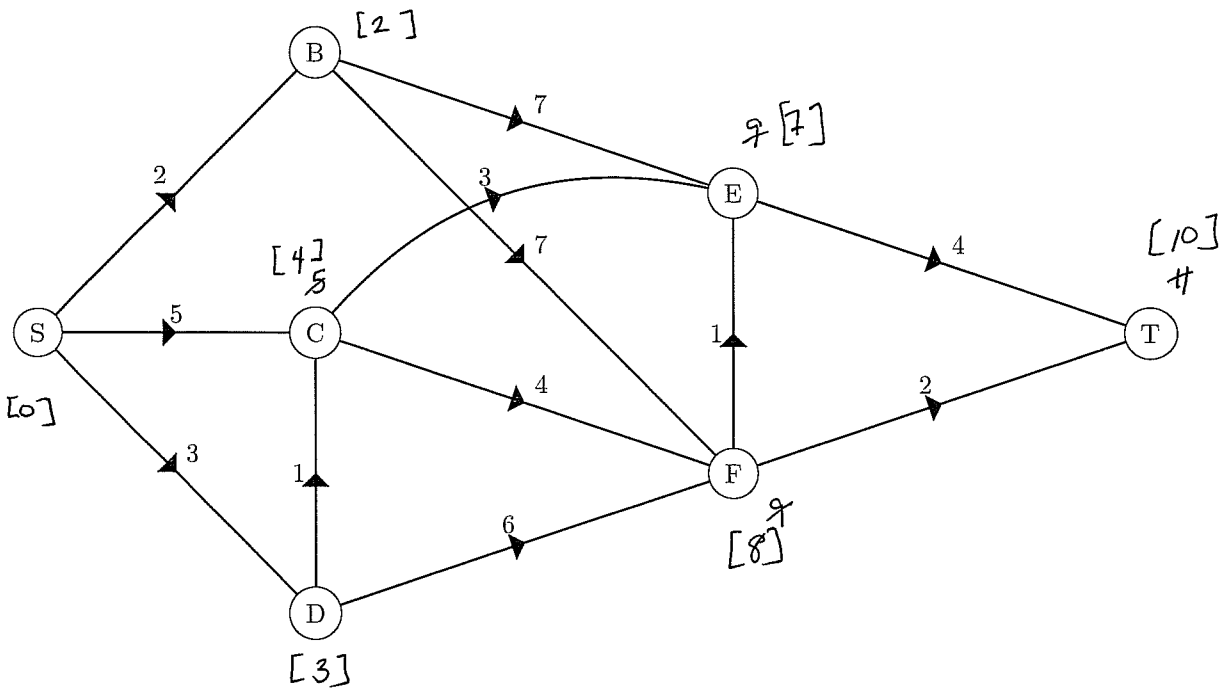
(There are 3 longest paths.)

Complete the following table according to scheduling the events represented in the above graph:

E-Earliest start time L-Latest start time F-Float time

	SA	SB	SC	SD	SF	AF	BE	CE	DH	EH	FG	GT	HT
E	0	0	0	0	0	1	3	2	4	5	4	6	8
L	4	0	0	0	3	5	3	2	4	5	7	9	8
F	4	0	0	0	3	4	0	0	0	0	3	3	0

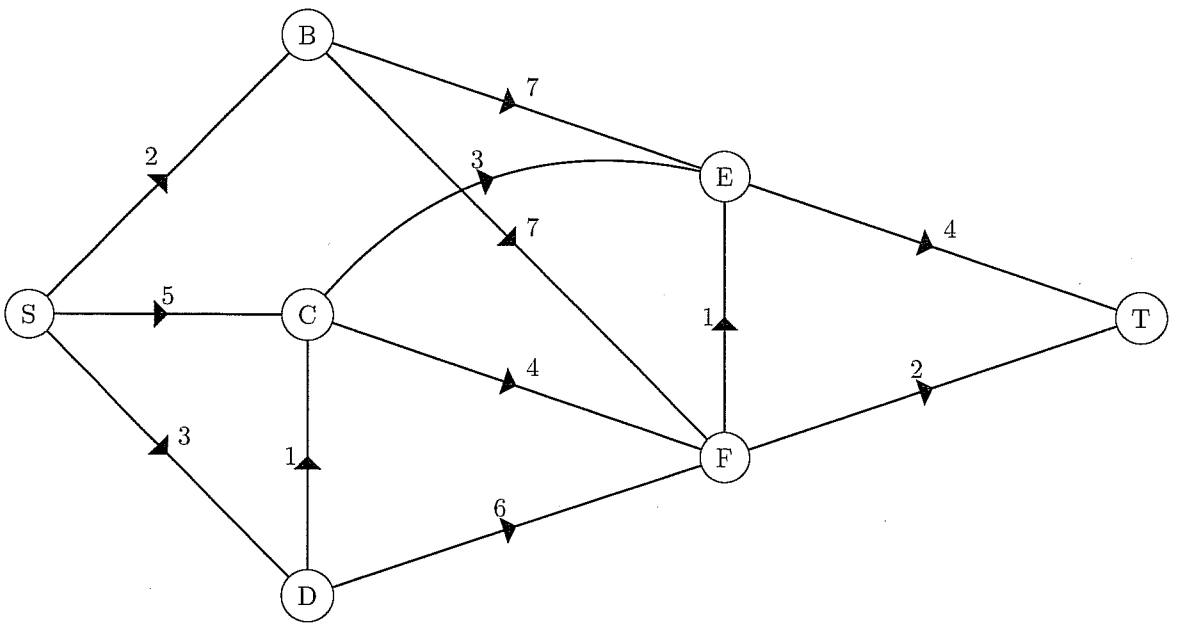
In the following weighted digraph, find the shortest path from node S to node T.



The shortest path has weight 10.

S → D → C → F → T

In the following weighted digraph, find the longest path from node S to node T.



S	B	C	D	E	F	T
—	S 2	SD 5 4	S 3	B C F 9 8 10	B C D 9 9 9	E F 14 11

[0] [2] [5] [3] [10] [9] [14]

The longest path has weight 11.

There are 3 longest paths

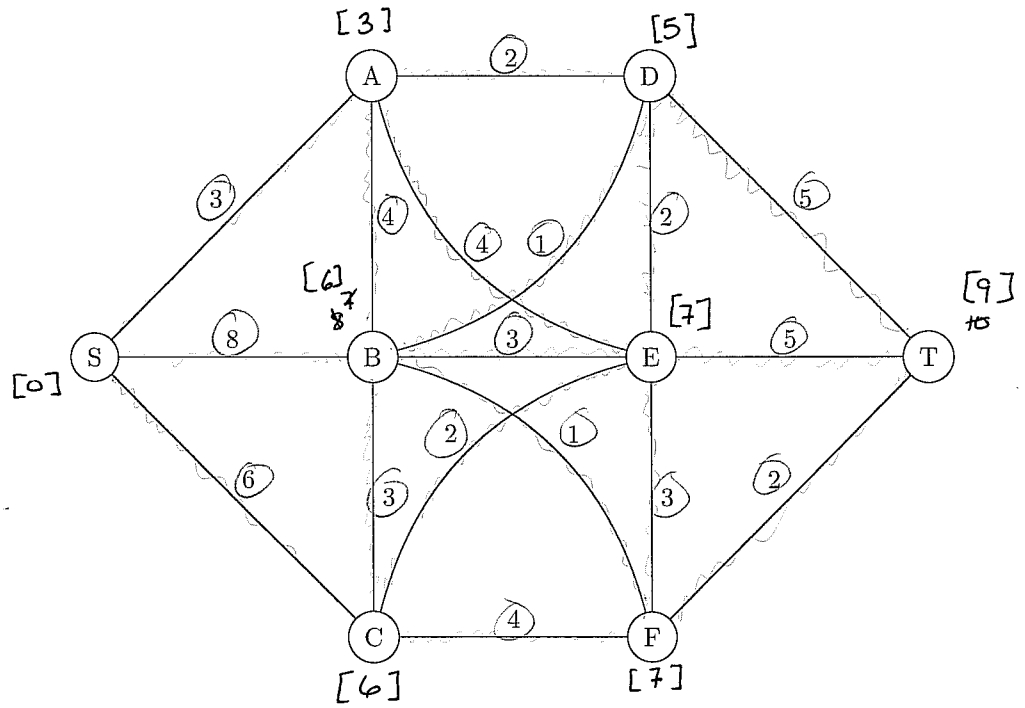
S → B
S → C → F → E → T
S → D

Complete the following table according to scheduling the events represented in the above graph:

E-Earliest start time L-Latest start time F-Float time

	SB	SC	SD	BE	BF	CE	CF	DC	DF	ET	FE	FT
E	0	0	0	2	2	5	5	3	3	10	9	9
L	0	0	0	3	2	7	5	4	3	10	9	12
F	0	0	0	1	0	2	0	1	0	0	0	3

In the following weighted digraph, find the shortest path from node S to node T.



The shortest path from S to T is of weight 9:

$$S \to A \to D \to B \to F \to T$$

To solve the Chinese Postman Problem, we find the Semi-Eulerian trail from S to T, then return to S via the shortest path:

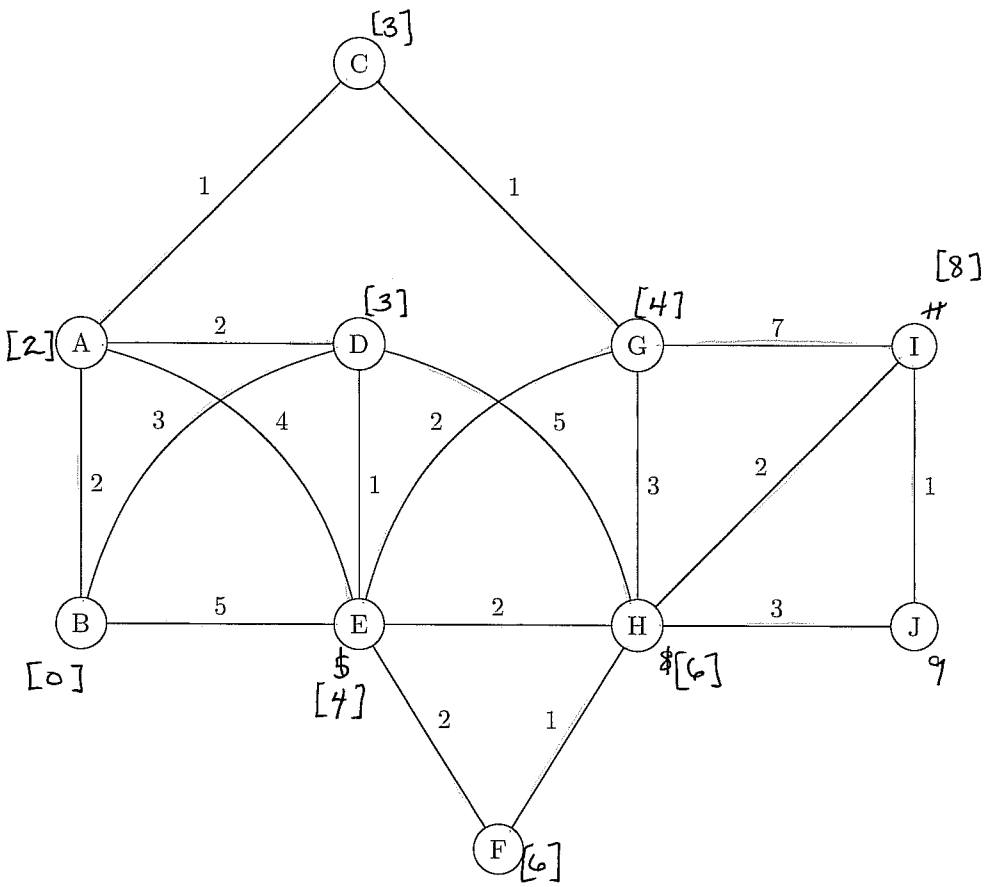
S-A-B-S-C-B-D-A-E-B-F-C-E-F-T-E-D-T
-F-B-D-A-S.

The weight of the postmans Route is (total of all edges in Graph) + (weight of shortest path)

$$= 58 + 9 = 67.$$

Solve the Chinese postman problem for the above graph.

Solve the Chinese postman problem for the following weighted digraph.



Both B and I are vertices of odd degree; all other vertex degrees are even. Hence the graph is semi Eulerian. We start by finding the shortest path from B to I.

The shortest path is B-D-E-H-I of weight 8

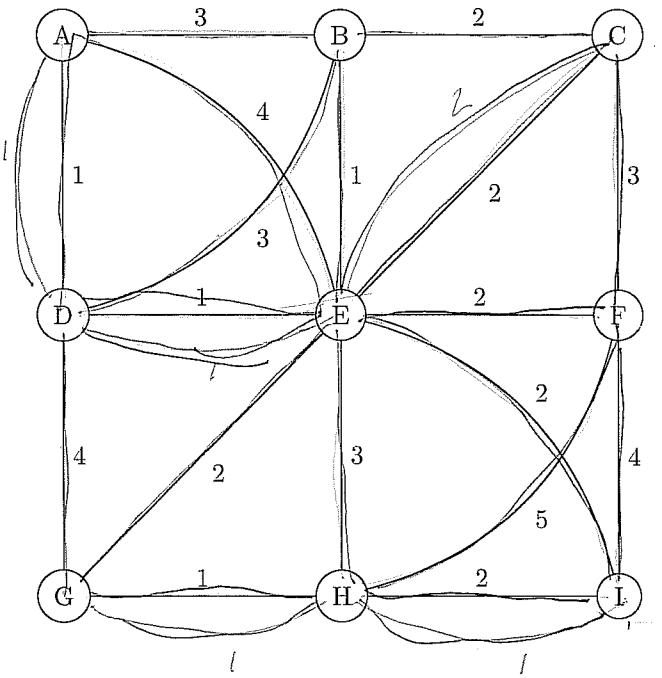
Hence the length of the Postmans total route is 55

A possible route is:

B-A-E-B-D-A-C-G-E-D-H-E-F-H-G-I-H-J-I

H-E-D-B

Solve the Chinese postman problem for the following weighted digraph.



We start by noticing that the nodes A, C, G and I all have odd degree. Hence the solution must take into account pairs of shortest paths, and choose the pair with the smallest total

AC → A-D-E-C of weight 4
GI → G-H-I of weight 3
} This pair has smallest total weight.

AG → A-D-E-G of weight 4
CI → C-E-I of weight 4

AI → A-D-E-I of weight 4
CG → C-E-G of weight 4

A walk could be:

C-B-A-D-E-C-F-I-H-G-H-I-E-F-H-E-G-D-E
-A-D-B-E-C

of weight 52.