MATH 2130 Tutorial 4

1. Find all unit tangent vectors to the curve $x^2 + z^2 = 4$, $x + y = 1$ at the point $(\sqrt{2}, 1 - \sqrt{2}, \sqrt{2})$.

2. Find the unit tangent vector to the curve $x = t^2$, $y = 3t^3$, $z = 3t^2$ at the origin.

3. Find the angle between the tangent vectors to the curves

$$x^2 + y = z + 4, \quad x + 2y = 5, \qquad \text{and} \qquad x + y^2 = 5, \quad 2x + 3y + 4z = 4$$

at the point of intersection of the curves.

4. Find the length of the curve

$$x = t + 1, \quad y = 2t^{3/2} - 3, \quad z = 4t - 2$$

between the points $(2, -1, 2)$ and $(1, -3, -2)$.

5. Show that it is impossible for the length of a curve joining the points $(1, -2, 3)$ and $(0, 4, 10)$ to be equal to 9.

6. Set up, but do not evaluate a definite integral, to find the length of the curve

$$x^2 + y^2 = z^2 - 4, \quad x + y = 4$$

joining the points $(4, 0, 2\sqrt{5})$ and $(2, 2, 2\sqrt{3})$. Simplify the integrand as much as possible.

Answers:

1. $\pm(-\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})/\sqrt{3}$

2. $(\hat{\mathbf{i}} + 3\hat{\mathbf{k}})/\sqrt{10}$

3. $\text{Cos}^{-1}\left(\dfrac{-21}{\sqrt{14}\sqrt{297}}\right)$

4. $2(26\sqrt{26} - 17\sqrt{17})/27$

6. $2\displaystyle\int_2^4 \sqrt{\frac{t^2 - 4t + 7}{t^2 - 4t + 10}}\,dt$