MATH 2500 Assignment #1

Due: January 29, 2014, Before Class (12:30)

Reminder: all assignments *must* be accompanied by a signed copy of the honesty declaration available on my website.

Assignments are to be handed in on $8\frac{1}{2} \times 11$ paper, single sided, no ragged edges, stapled in the top left hand corner with the honesty declaration as the first page.

1. Use induction to prove the following

$$2 + 5 + 8 + 11 + \cdots + (12n - 1) = 2n(12n + 1).$$

2. The Fibonacci numbers are defined by

$$f_{n+1} = f_n + f_{n-1} \; ; \; f_1 = f_2 = 1.$$

   Use induction to prove that f_{4n} is divisible by 3.

3. Use the Euclidean Algorithm to find $(5556, 1062)$ and then find integer values for x and y such that $5556x + 1062y = (5556, 1062)$.

4. (a) Find the prime power decomposition of the following numbers:
      i. 147420
      ii. 184800
      iii. 282744.

   (b) Use the results of part (a) to find:
      i. $(147420, 184800)$
      ii. $(147420, 282744)$
      iii. $(184800, 282744)$.

5. Prove (using the appropriate definitions, theorems and/or lemmas from the text) :

   (a) If $a \mid b$ and $c \mid d$ then $ab \mid cd$.

   (b) If $a \mid c$ and $b \mid c$ and $(a, b) = d$, then $ab \mid cd$.