Math 2500 Assignment #2 W-2014

1(a) $1239x + 553y = 21$

$1239 = 553(2) + 133$

$553 = 133(4) + 21$

$133 = 21(6) + 7$

$21 = 7(3) + 0$ So $(1239, 553) = 7$ & $7 \mid 21$

$7 = 133 + 21(-6) = 133 + [553 + 133(-4)](-6)$

$= 553(-6) + 133(25) = 553(-6) + [1239 + 553(-2)](25)$

$= 1239(25) + 553(-56)$

Hence $1239(25) + 553(-56) = 7$

$1239(75) + 553(-168) = 21$

all solutions:

$x = 75 + 79t$ $\quad (79 = 553/7)$

$y = -168 - 177t$ $\quad (177 = 1239/7)$

(b) $572x + 1309y = 21$

$1309 = 572(2) + 165$

$572 = 165(3) + 77$

$165 = 77(2) + 11$

$77 = 11(7) + 0$

So $(572, 1309) = 11$ but $11 \nmid 21$, hence no solutions.

Q1 (c) $978x + 1113y = 21$

$$1113 = 978(1) + 135$$
$$978 = 135(7) + 33$$
$$135 = 33(4) + 3$$
$$33 = 3(11) + 0$$

So $(978, 1113) = 3$ & $3 \mid 21$.

$$3 = 135 + 33(-4) = 135 + [978 + 135(-7)](-4)$$
$$= 978(-4) + 135(29) = 978(-4) + [1113 + 978(-1)](29)$$
$$= 1113(29) + 978(-33)$$

Hence $978(-33) + 1113(29) = 3$

$978(-231) + 1113(203) = 21$

all solutions: $x = -231 + 371t$ $\quad (371 = 1113/3)$

$y = 203 - 326t$ $\quad (326 = 978/3)$.

Q2 (a) $877x \equiv 146 \pmod{2775}$

$$\begin{array}{l|l}
2775 = 877(3) + 144 & \\
877 = 144(6) + 13 & 1 = 144 + 13(-11) \\
144 = 13(11) + 1 & \quad = 877(-11) + 144(67) \\
13 = 1(13) + 0 & \quad = 2775(67) + 877(-212)
\end{array}$$

(So an inverse of 877 mod 2775 is -212)

$$(-212)\,877x \equiv (-212)\,146 \mod 2775$$
$$x \equiv -30952 \equiv 2348 \mod 2775$$

The solution is 2348.

(b) $2980x \equiv 1262 \pmod{1288}$

$404x \equiv 1262 \pmod{1288}$

$$1288 = 404(3) + 76$$
$$404 = 76(5) + 24$$
$$76 = 24(3) + 4$$
$$24 = 4(6) + 0$$

So $(1288, 404) = 4$ and $4 \nmid 1262$

So there are no solutions.

2(c) $759x \equiv 102 \mod 1638$

$$1638 = 759(2) + 120$$
$$759 = 120(6) + 39$$
$$120 = 39(3) + 3$$
$$39 = 3(13) + 0$$

$(759, 1638) = 3$ and $3 \mid 102$.

$$3 = 120 + 39(-3)$$
$$= 759(-3) + 120(19)$$
$$= 1638(19) + 759(-41)$$

$$(-41)759x \equiv (-41)102 \mod 1638$$
$$3x \equiv -4182 \mod 1638$$
$$x \equiv -1394 \equiv 244 \mod 546$$

The solutions are 244, 790, 1336

3(a) $x \equiv 4 \pmod{17}$

$x \equiv 37 \pmod{73}$

$x \equiv 57 \pmod{75}$

$x = 4 + 17k_1$

$4 + 17k_1 \equiv 37 \bmod 73$

$17k_1 \equiv 33 \bmod 73$

$(-30)17k_1 \equiv (-30)33 \bmod 73$

$k_1 \equiv -990 \equiv 32 \bmod 73$

$k_1 = 32 + 73k_2$

$73 = 17(4) + 5$

$17 = 5(3) + 2$

$5 = 2(2) + 1$

$2 = 1(2) + 0$

$1 = 5 + 2(-2)$

$= 17(-2) + 5(7)$

$= 73(7) + 17(-30)$

$x = 4 + 17(32 + 73k_2)$

$= 548 + 1241k_2$

$548 + 1241k_2 \equiv 57 \bmod 75$

$41k_2 \equiv -491 \equiv 34 \bmod 75$

$75 = 41(1) + 34$

$41 = 34(1) + 7$

$34 = 7(4) + 6$

$7 = 6(1) + 1$

$6 = 1(6) + 0$

$1 = 7 + 6(-1)$

$= 34(-1) + 7(5)$

$= 41(5) + 34(-6)$

$= 75(-6) + 41(11)$

$(11)41k_2 \equiv (11)34 \bmod 75$

$k_2 \equiv 374 \equiv 74 \bmod 75$

$k_2 = 74 + 75k_3$

$x = 548 + 1241(74 + 75k_3)$

$= 92382 + 93075k_2$

$x \equiv 92382 \pmod{93075}$ Note: This should be in least residue.

3(b) $x \equiv 122 \bmod 169$

$x \equiv 31 \bmod 221$

$x = 122 + 169k_1$

$122 + 169k_1 \equiv 31 \bmod 221$

$169k_1 \equiv -91 \bmod 221$

$169k_1 \equiv 130 \bmod 221$

$221 = 169(1) + 52$ $\qquad$ $(169, 221) = 13$, $13 \mid 130$.

$169 = 52(3) + 13$ $\qquad$ $13 = 169 + 52(-3)$

$52 = 13(4) + 0$ $\qquad$ $= 221(-3) + 169(4)$

$(4)169k_1 \equiv (4)130 \bmod 221$

$13k_1 \equiv 520 \bmod 221$

$13k_1 \equiv 78 \bmod 221$

$k_1 \equiv 6 \bmod 17$

$k_1 = 6 + 17k_2$

$x = 122 + 169(6 + 17k_2) = 1136 + 2873k_2$

So $x \equiv 1136 \pmod{2873}$

(c) $x \equiv 51 \pmod{112}$ $\qquad$ $x = 51 + 112k_1$

$x \equiv 89 \pmod{147}$

$147 = 112(1) + 35$

$51 + 112k_1 \equiv 89 \bmod 147$ $\qquad$ $112 = 35(3) + 7$

$112k_1 \equiv 38 \bmod 147$ $\qquad$ $35 = 7(5) + 0$

$(112, 147) = 7$ and $7 \nmid 38$ so no solutions.

Q4 If $2413 \equiv 4142 \mod m$, then $m \mid 4142-2413$ or $m \mid 1729$, and m is positive.
Now $1729 = 7 \times 13 \times 19$
So possible values of m are:
1, 7, 13, 19, 91, 133, 247 and 1729.

Q5(a) We want to show that $n^5 \equiv n \mod 10$
So now $n \equiv 0, 1, 2, 3, 4, 5, 6, 7, 8$ or $9 \mod 10$

n	n^5	$n^5 \mod 10$	n	n^5	$n^5 \mod 10$
0	0	0	5	3125	5
1	1	1	6	7776	6
2	32	2	7	16807	7
3	243	3	8	32768	8
4	1024	4	9	59049	9

Hence for any n, $n^5 \equiv n \mod 10$.

(b) If $n \equiv 0 \pmod{12}$ then $12 \mid n$,
If $n \equiv 2 \pmod{12}$ then $2 \mid n$, If $n \equiv 3 \pmod{12}$ then $3 \mid n$,
If $n \equiv 4 \pmod{12}$ then $4 \mid n$, If $n \equiv 6 \pmod{12}$ then $6 \mid n$,
If $n \equiv 8 \pmod{12}$ then $4 \mid n$, If $n \equiv 9 \pmod{12}$ then $3 \mid n$,
If $n \equiv 10 \pmod{12}$ then $2 \mid n$.

So if n is not 2 or 3, and is not congruent to 1, 5, 7 or 11 (mod 12), then n is composite.
Hence if p is prime (not 2 or 3), it is congruent to 1, 5, 7 or 11 (mod 12).