MATH 2500 Assignment #4

Due: April 2, 2014, Before Class (12:30)

Reminder: all assignments *must* be accompanied by an honesty declaration available on my website.

1. (a) How many primitive roots does 101 have?
   (b) What are the possible orders of an element modulo 101?
   (c) Show that 2 is a primitive root of 101.
   (d) Find 4 other primitive roots. (Note, these should be in least residue.)
   (e) Given that $2^{14} = 22 + 101(162)$, what is the order of $22 \pmod{101}$? Of $44 \pmod{101}$? Of $88 \pmod{101}$?

2. Which of the following quadratic congruences have solutions?
   (You do NOT need to find solutions if they exist.)
   (a) $2x^2 + 7x + 4 \equiv 0 \pmod{61}$
   (b) $7x^2 + x + 17 \equiv 0 \pmod{73}$
   (c) $x^2 - 77x + 38 \equiv 0 \pmod{91}$ (Hint: be careful, 91 is NOT prime.)

3. Solve the following Legendre symbols:
   (a) $\left(\dfrac{4014}{6551}\right)$ (b) $\left(\dfrac{3003}{8053}\right)$ (c) $\left(\dfrac{3371}{7331}\right)$

4. Which of $3, 5, 7, 11$ are primitive roots of 1187?
   [Hint: Consider Euler's Criterion.]

5. (a) Given p an odd prime, $p \neq 3$, find a rule for the value of the Legendre symbol $\left(\dfrac{3}{p}\right)$.
   (Hint: your answer should be in modulo 12.)
   (b) Suppose that 3 is a primitive root of p, what are the possible values of $p \pmod{12}$?
   (c) Given p an odd prime, $p \neq 3$, find a rule for the value of the Legendre symbol $\left(\dfrac{6}{p}\right)$.
   (Hint: your answer should be in modulo 24.)
   (d) Suppose that 6 is a primitive root of p, what can be said about $p \pmod{24}$?
   (e) What can be said about p if both 3 and 6 are primitive roots of p?
   (f) Is it possible for all of 2, 3 and 6 to be primitive roots of the same prime p? (Explain.)
   (g) If g and h are both primitive roots of a prime p, is it possible for gh (or the least residue of gh) to be a primitive root? (Fully justify your answer.)