UNIVERSITY OF MANITOBA

DATE: December 11, 2010   FINAL EXAMINATION
                                                    TITLE PAGE
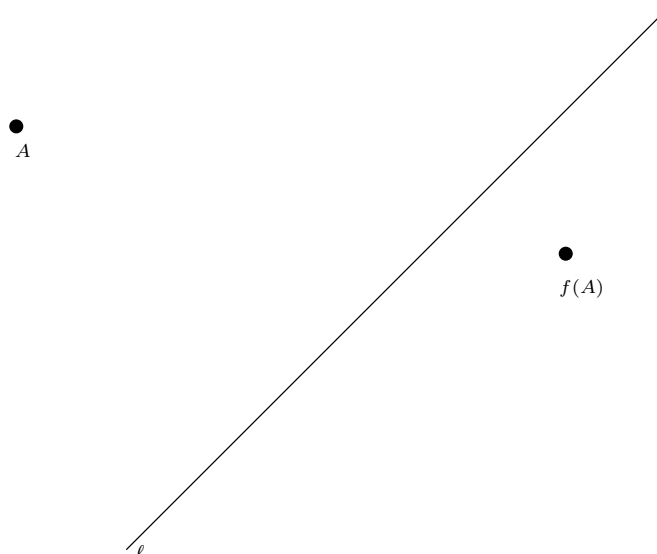COURSE: MATH\FA 1020   TIME: 2 hours
EXAMINATION: Math in Art   EXAMINER: M. Davidson

FAMILY NAME: (Print in ink) _____

GIVEN NAME(S): (Print in ink) _____

STUDENT NUMBER: _____

SEAT NUMBER: _____

SIGNATURE: (in ink) _____
                  (I understand that cheating is a serious offense)

INSTRUCTIONS TO STUDENTS:

This is a 2 hour exam. **Please show
your work clearly.**

A compass and straight edge (ruler) are
required for this exam.

No texts, notes, or other similar aids are
permitted. There are no
calculators,cellphones or electronic
translators permitted.

This exam has a title page and 8 pages of
questions. Please check that you have all
the pages.

The value of each question is indicated in
the lefthand margin beside the statement
of the question. The total value of all
questions is 70 points.

**Answer all questions on the exam
paper** in the space provided beneath the
question. If you need more room, you
may continue your work on the reverse
side of the page, but CLEARLY
INDICATE that your work is continued.

Question	Points	Score
1	10	
2	8	
3	8	
4	12	
5	9	
6	8	
7	9	
8	6	
Total:	70	

Important: "Construct" means "construct using an unmarked ruler and compass." When you use a compass, show the (intermediate) circular arcs you draw in your constructions (do not erase them). Use words to describe **BRIEFLY** what you have done.

[10]   1.   (a) Recall that a dilative reflection is a composition of a central similarity followed by a reflection with respect to a line passing through the center of the central similarity. Given that f is a dilative reflection, (where ℓ is the line of reflection), find C, the center of the central similarity.

(b) Construct a golden rectangle having the given segment as one of its (shorter) sides.

[8] 2. (a) List the symmetries of the following object:

(b) List the symmetries of the following object (it is a frieze pattern, it continues indefinitely on either side):

(c) Draw an object that has exactly 5 symmetries (including the identity).

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[8]   3. In the figures below labeled Step 1 and Step 2 we depict the first two steps in the construction of a fractal **F**.

   (a) In the square labeled Step 3, draw the next step in the construction of the fractal.

   (b) The fractal **F** will be constructed after infinitely many steps. Find a central similarity (stretching factor not equal to 1) which maps **F** into itself. (To get full marks, you should indicate in Step 3 the point the is the center of the central similarity and you need to state a specific number for the stretching factor of the similarity.
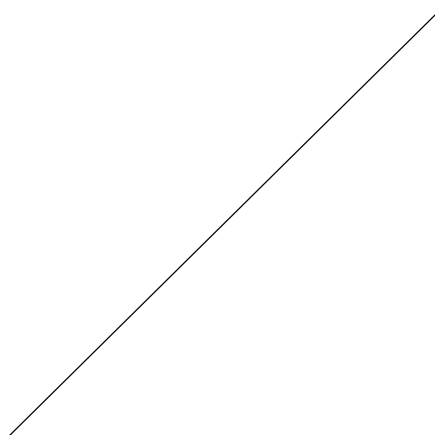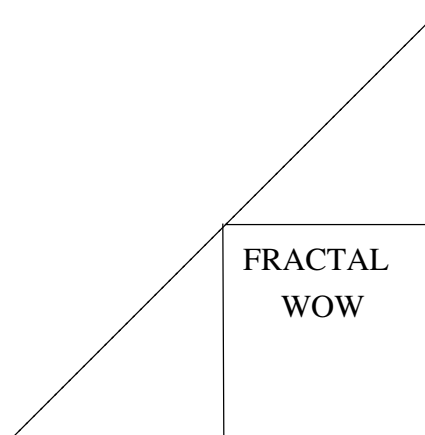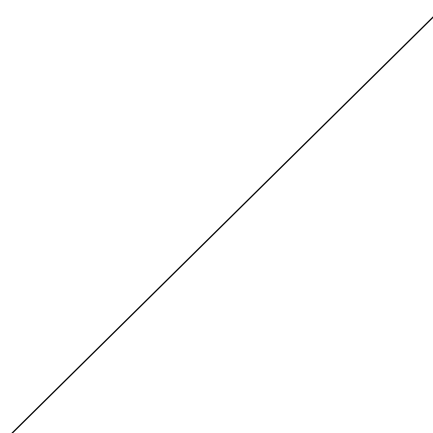
FRACTAL

WOW

Step 1   Step 2

Step 3

[12]   4.   (a)   Fill in the following table with the appropriate information about the Platonic solids.

Name of Solid	number of Faces	number of Edges	number of Vertices
tetrahedron	4		
	20	30	
		12	6
cube	6		
dodecahedron		30	

(b)   Consider the surface of the dodecahedron, as a two-manifold.

   i. What is the genus of the surface of the dodecahedron?

   ii. What is the Euler Characteristic of the surface of the dodecahedron?

(c)   Given the following figure (Note: The black parts indicate that the hole goes right through the object):
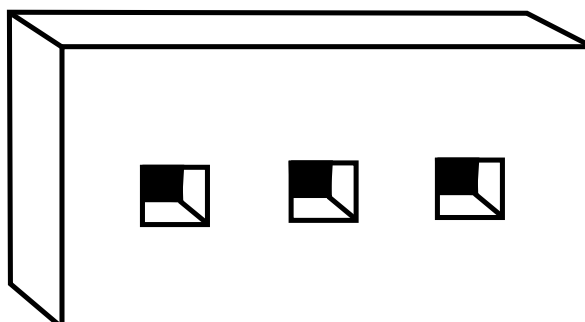
   i. What is the genus of the surface of the figure?

   ii. What is the Euler Characteristic of the surface of the figure?

[9] 5. Given below is the set up for the construction of a hyperbola. Sketch the hyperbola that passes through points A and B by finding 12 more points on the hyperbola (coming from 3 circles).
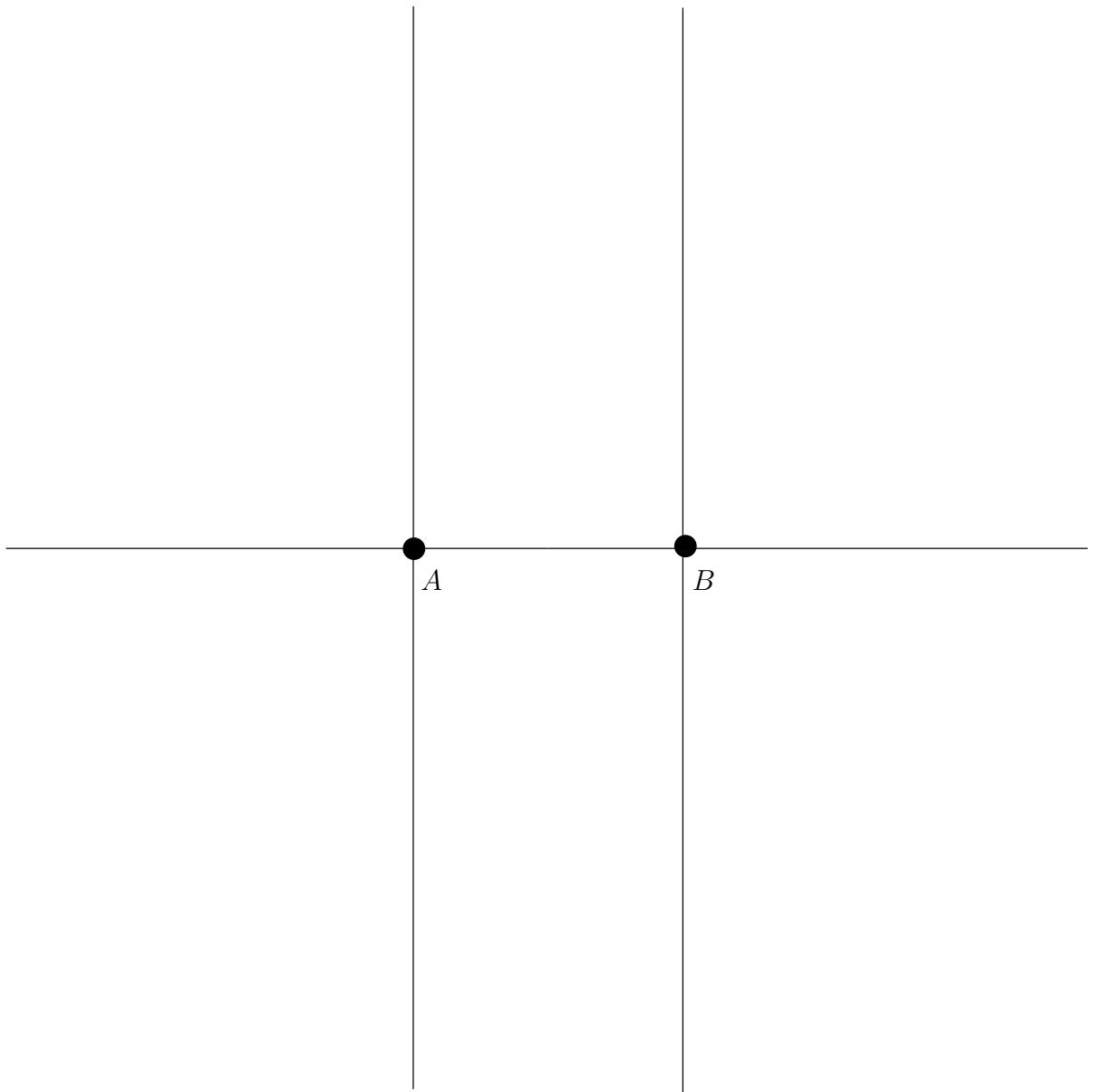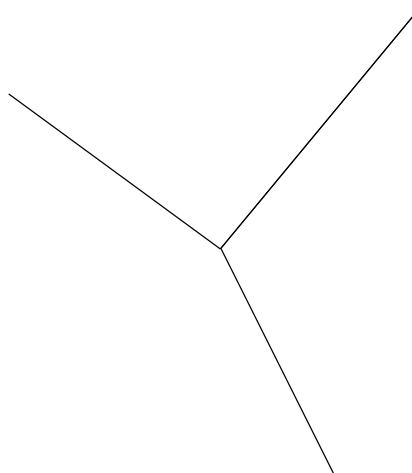
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[8] 6. Below is a portion of a drawing of a **cube** in 3-point perspective. One vertex of the cube is given, along with the three edges which meet it. One of the vanishing points, V_3, is given, and the other two (not given) are on the horizon h.

(a) Find the two missing vanishing points, V_1 and V_2.

(b) Draw (construct) **all** remaining edges of the cube.

h

• V_3

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[9] 7. Below is a hyperbolic plane, having center O.

   (a) Find (construct) the hyperbolic line that passes through A and intersects ℓ at the point B.

   (b) Find (construct) one hyperbolic line that passes through A and is parallel to ℓ.
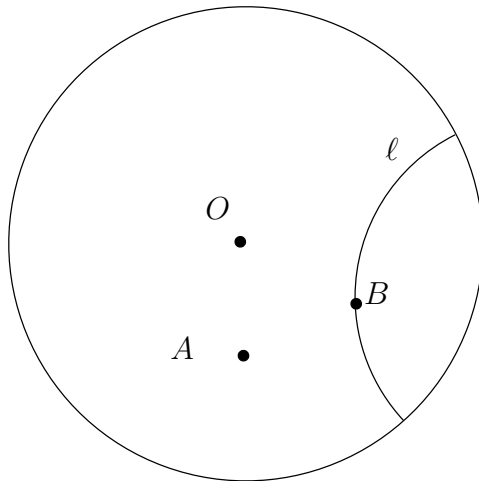
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[6] 8. (a) Identify which letters and numbers are mutually homotopic in the following phrase:

<div align="center">

BORUS : 555 - 4982

</div>

(b) Show that the two spaces depicted below (the glasses and the scissors) are homotopic by drawing at least three intermediate steps in the homotopy.