UNIVERSITY OF MANITOBA

DATE: April 14, 2012 — FINAL EXAMINATION
TITLE PAGE

COURSE: MATH 2400 — TIME: 3 hours

EXAMINATION: Graph Theory — EXAMINER: M. Davidson

FAMILY NAME: (Print in ink) _____

GIVEN NAME(S): (Print in ink) _____

STUDENT NUMBER: _____

SEAT NUMBER: _____

SIGNATURE: (in ink) _____
(I understand that cheating is a serious offense)

INSTRUCTIONS TO STUDENTS:

This is a 3 hour exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page and 13 pages of questions. Please check that you have all the pages.

The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 180 points.

Answer questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	32	
2	14	
3	10	
4	16	
5	20	
6	10	
7	14	
8	16	
9	8	
10	14	
11	14	
12	12	
Total:	180	

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[32] 1. For each of the following, draw a graph (or a digraph) on n vertices that satisfies the given properties if one exists. If no such graph (or digraph) exists, clearly show why not.

(a) $n = 6$; connected, (at least one) vetrex of degree 4, every edge is a bridge.

(b) $n = 6$; a tournament that is not strongly connected, but has no vertices of outdegree 0, and no vertices of outdegree 5.

(c) $n = 6$; Simple, having degree sequence $\{0, 2, 2, 2, 4\}$.

(d) $n = 7$; Hamiltonian, but not Eulerian, nor semi-Eulerian.

(e) $n = 7$; 5-regular.

(f) $n = 6$; Simple, connected, nonplanar, and no cycles of odd length.

(g) $n = 7$; A complete bipartite graph that is also a tree.

(h) $n = 7$; isomorphic to its complement.

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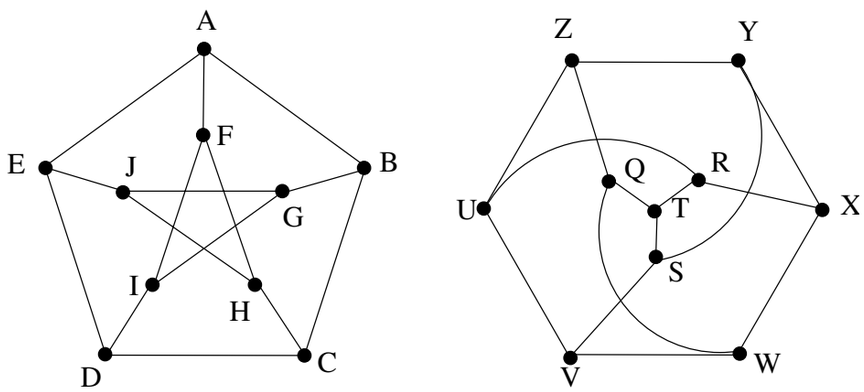
[14] 2. The following are two common drawings of the Petersen graph.



(a) Show that the above graphs are isomorphic (give the isomorphism).

(b) What is the vertex connectivity ($\kappa(G)$) of the Petersen graph? (Give the appropriate vertex cutset, from either graph.)

(c) What is the edge connectivity ($\lambda(G)$) of the Petersen graph? (Give the appropriate edge cutset, from either graph.)

(d) Give three (3) cutsets of different sizes, all different from $\lambda(G)$.

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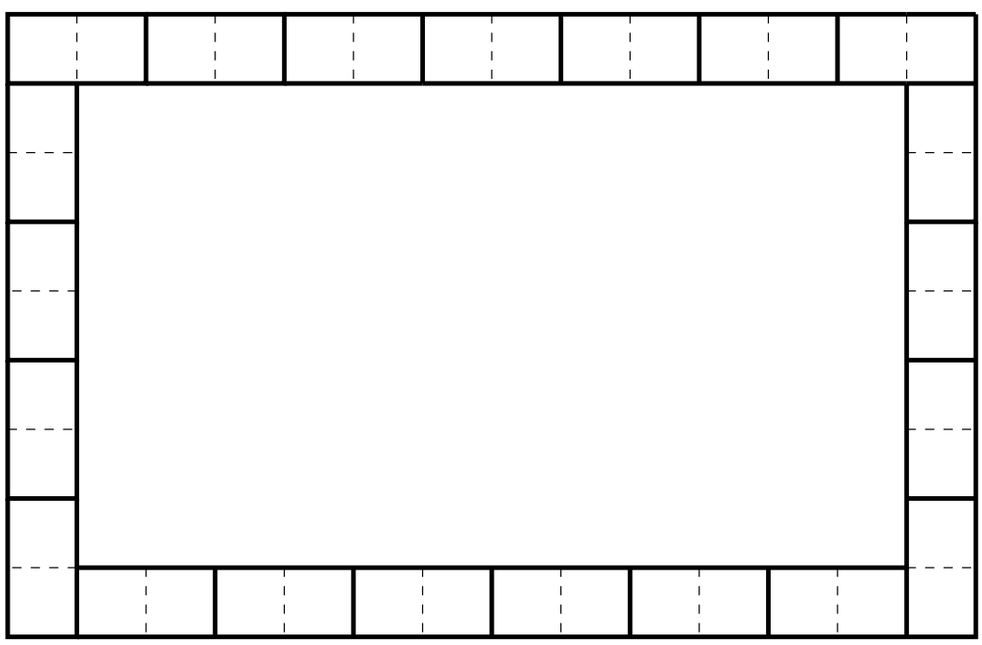
[10] 3. (a) Below are the dominoes from a double 6 set of dominoes with the doubles removed. Fill in the circuit of dominoes so that two dominoes are side by side in the circuit if and only if the adjacent squares have the same number.

0 / 1	0 / 2	0 / 3	0 / 4	0 / 5	0 / 6	1 / 2
1 / 3	1 / 4	1 / 5	1 / 6	2 / 3	2 / 4	2 / 5
2 / 6	3 / 4	3 / 5	3 / 6	4 / 5	4 / 6	5 / 6



(b) Explain, in terms of graphs, what was found. For which types of dominoe sets could a similar circuit be found?

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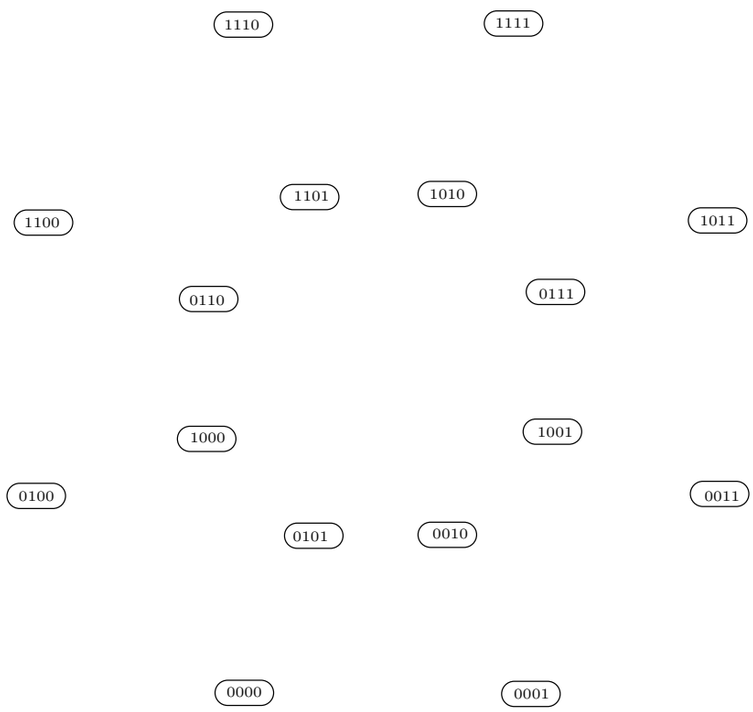
[16] 4. (a) On the following labelled vertices, draw all the edges of the cube graph Q_4.



(b) Give a Gray code on binary words of length 4, and indicate in the graph drawn above the edges associated with the Gray code. (You may do this by using a different colour, or by drawing a squiggle through the lines). What is now drawn on the above graph?

(c) Show that Q_4 is bipartite. (You may do this by listing the appropriate sets, or by drawing, say triangles and circles around appropriate vertices above.)

(d) Considering what was shown in part (c), what can you say about the length of a cycle in Q_4?

[20] 5. (a) Draw the tree having the following Prüfer sequences.

i. 1 1 4 2 2 2

ii. 1 2 1 2 1 2

iii. 1 1 8 8 1 2

(b) Which two of these trees are isomorphic? (You do not need to give the isomorphism.)

(c) Give a reason why the third tree is not isomorphic to the other two.

(d) i. What is the center / bicenter of tree (i)?

ii. What is the centroid / bicentroid of tree (i)?

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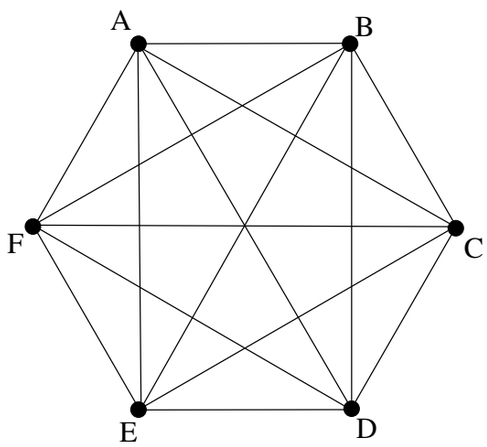
[10] 6. Consider the following weighted graph:



	A	B	C	D	E	F
A	–	6	2	5	4	9
B	6	–	3	5	5	2
C	2	3	–	7	8	2
D	5	5	7	–	9	6
E	4	5	8	9	–	7
F	9	2	2	6	7	–

(a) Find an lower bound for the solution to the travelling salesman problem by removing A.

(b) Find an lower bound for the solution to the travelling salesman problem by removing D.

(c) Which of the bounds found above is the better bound?

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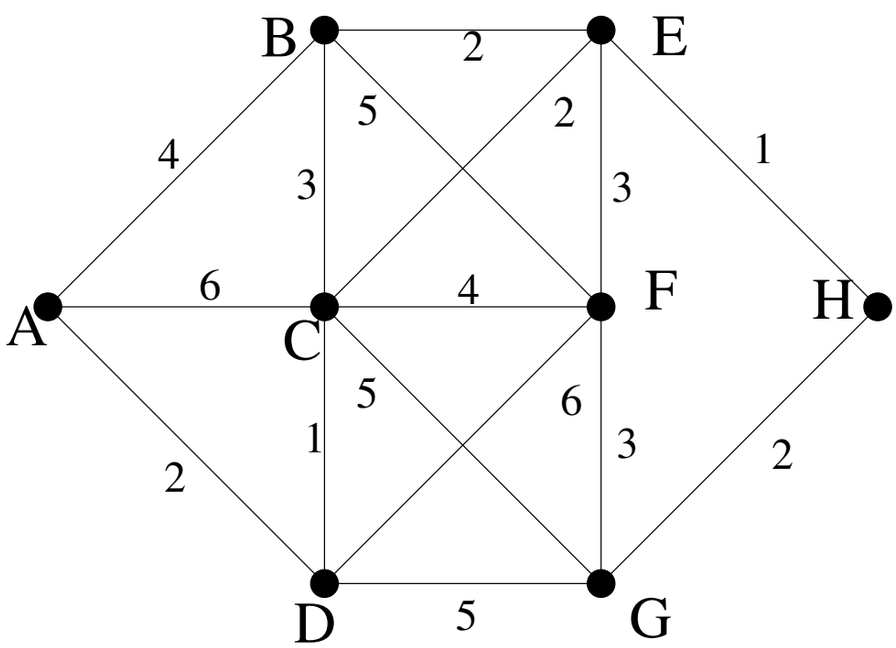
[14] 7. Solve the Chinese postman problem for the following graph. Show all steps for any shortest path algorithm used. Give the solution in term of an appropriate walk and its weight.



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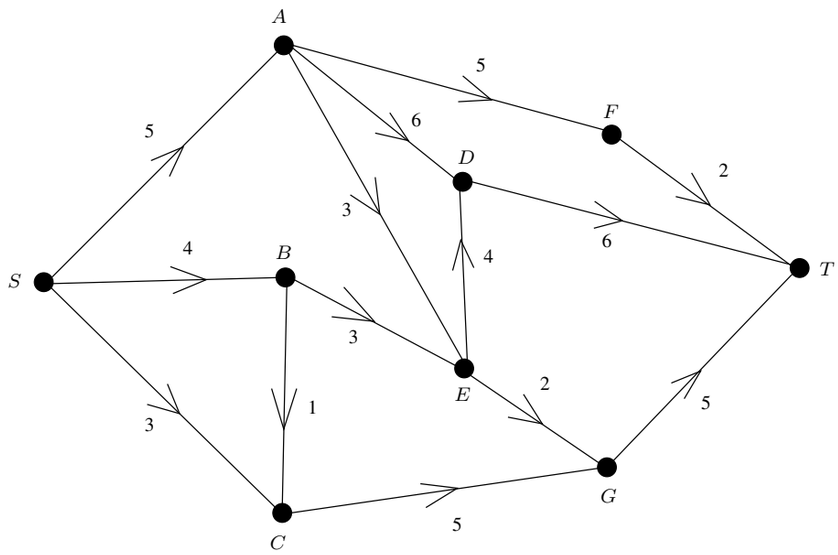
[16] 8. In the following weighted digraph:



(a) Find the longest path from S to T

(b) Complete the following table according to scheduling the events represented in the above graph:

E-Earliest start time L-Latest start time F-Float time

	SA	SB	SC	AD	AE	AF	BC	BE	CG	DT	ED	EG	FT	GT
E														
L														
F														

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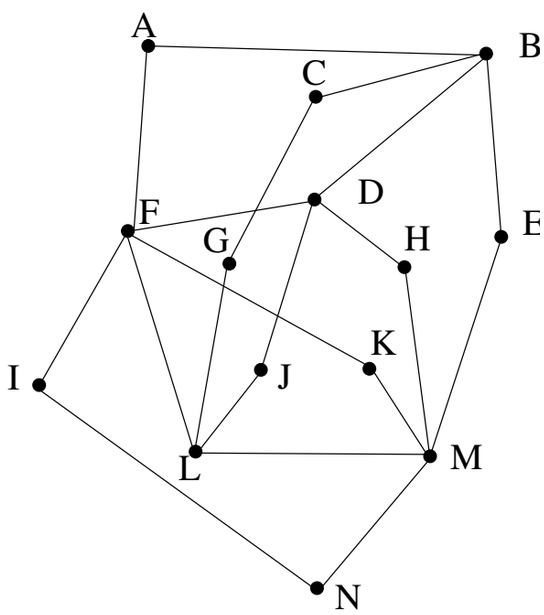
[8] 9. (a) State Kuratowski's Theorem.

(b) Show that the following graph is not planar. (Any theorem you use should be clearly stated.)



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[14] 10. (a) State Euler's formula, and the conditions under which it holds.

(b) Use the above to show that for a simple connected planar graph G having V-vertices, E-edges, then $E \leq 3V - 6$.

(c) Show that K_5 is not planar.

(d) Draw an example of a connected planar graph having 5 vertices and 10 edges (if one exists). How many faces does it have?

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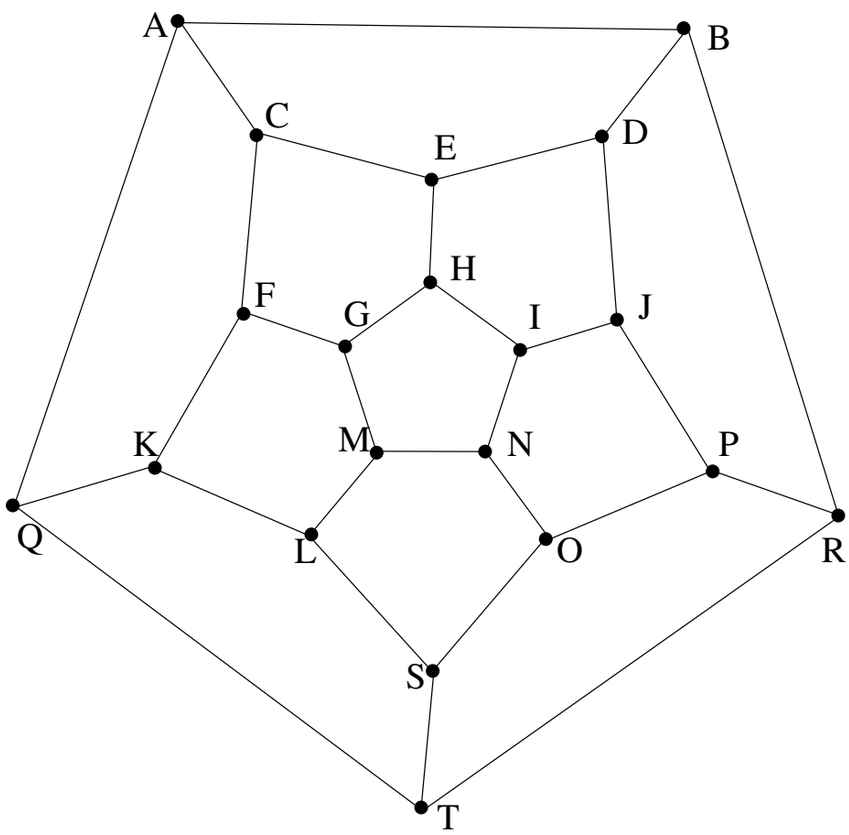
[14] 11. Below is a planar drawing of the dodecahedron graph G .



(a) Draw the dual of the dodecahedron G^*. (label the vertices of G^* with $f_1, f_2, \dots$)

(b) What is the name of G^*

(c) Find an edge cutset of G having 6 edges.

(d) List the edges in G^* which correspond to the edges listed in part (c). What do these edges form?

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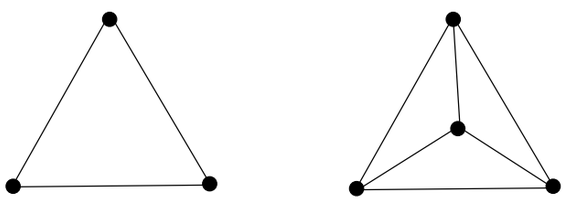
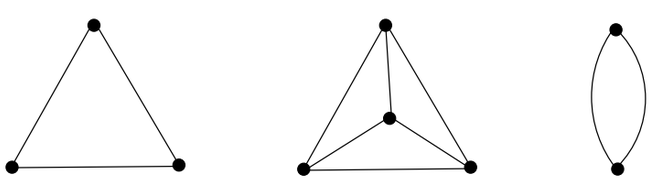
[12] 12. (a) Each of the following graphs is a planar drawing of a disconnected graph. For each of them, find the value of $V - E + F$. For each of them, give the number of components.

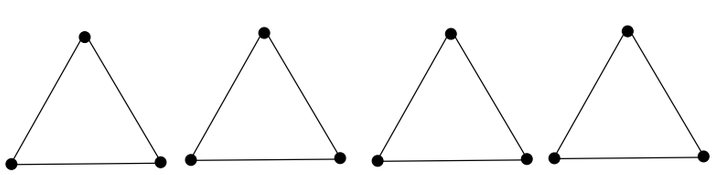
i.



ii.



iii.



(b) Can you find a relationship between the number of components and the value of $V - E + F$? (Perhaps consider the value of $V - E + F$ for connected planar graphs.)