UNIVERSITY OF MANITOBA

DATE: December 14, 2013 | FINAL EXAMINATION
TITLE PAGE

COURSE: MATH 2400 | TIME: 3 hours

EXAMINATION: Graph Theory | EXAMINER: M. Davidson

FAMILY NAME: (Print in ink) ____________________

GIVEN NAME(S): (Print in ink) ____________________

STUDENT NUMBER: __________

SEAT NUMBER: __________

SIGNATURE: (in ink) ____________________

(I understand that cheating is a serious offense)

INSTRUCTIONS TO STUDENTS:

This is a 3 hour exam. **Please show your work clearly.**

No texts, notes, or other aids are permitted. There are no calculators, cellphones or electronic translators permitted.

This exam has a title page and 12 pages of questions. Please check that you have all the pages.

The value of each question is indicated in the lefthand margin beside the statement of the question. The total value of all questions is 120 points.

Answer questions on the exam paper in the space provided beneath the question. If you need more room, you may continue your work on the reverse side of the page, but CLEARLY INDICATE that your work is continued.

Question	Points	Score
1	10	
2	12	
3	8	
4	12	
5	18	
6	14	
7	14	
8	8	
9	12	
10	12	
Total:	120	

---

[10] 1. Describe any **two (2)** of the following 'case studies'. Include the following:

- A description of the problem.
- A description of the graph used to solve the problem.
- A description of the solution to the problem with respect to the graph.

(a) Dominoes

(b) Gray Codes

(c) Rotating Drum problem

(d) Braced Rectangular Frameworks

(e) Chinese Postman problem

[12] 2. For each of the following, draw a graph (or a digraph) on n vertices that satisfies the given properties if one exists. If no such graph (or digraph) exists, clearly show why not.

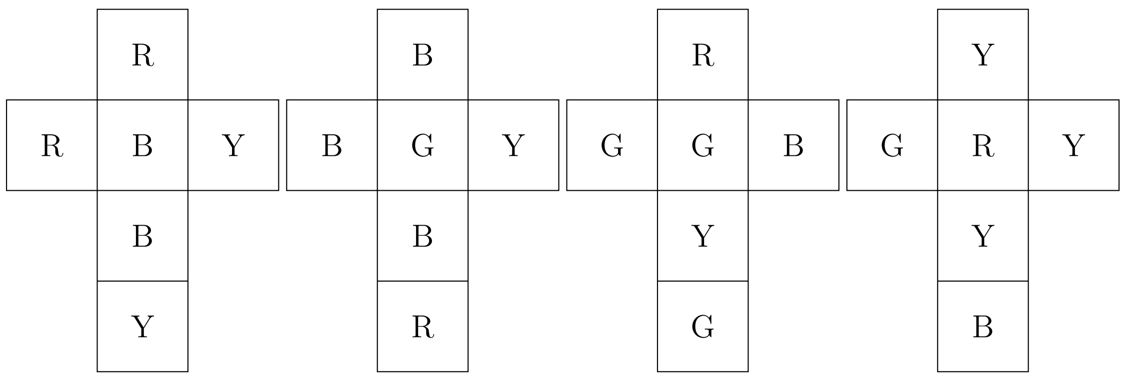
(a) $n = 6$; isomorphic to its complement.

(b) $n = 7$; A complete bipartite graph that is also a tree.

(c) $n = 6$; Simple, connected, nonplanar, and no cycles of odd length.

(d) $n = 7$; 5-regular.

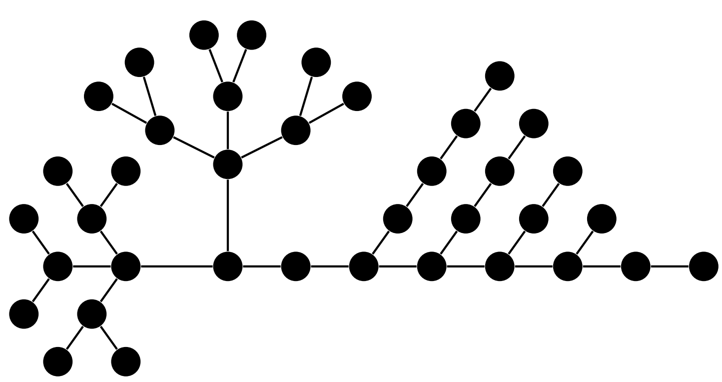
[8] 3. Solve the following four cube problem. Include the graph of the problem, as well as the graphs that give the solution to the problem.



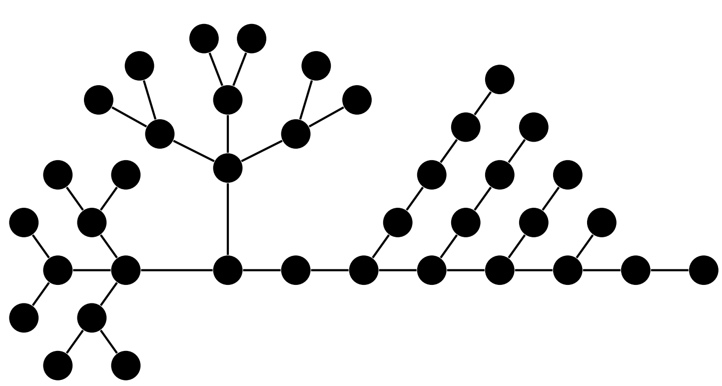
cube 1 cube 2 cube 3 cube 4

[12] 4. (a) Prove that if G is a graph such that there is exactly one path between every pair of vertices, then G is a tree.

(b) Find the center/bicenter of the following tree.



(c) Find the centroid/bicentroid of the following tree.



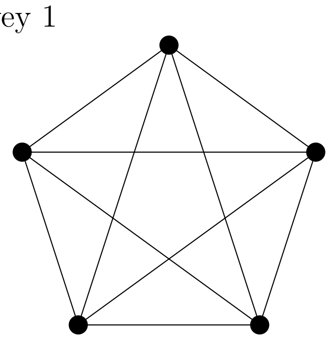
---

[18] 5. Consider the following results of a survey of fruits where the preferred choice is underlined:

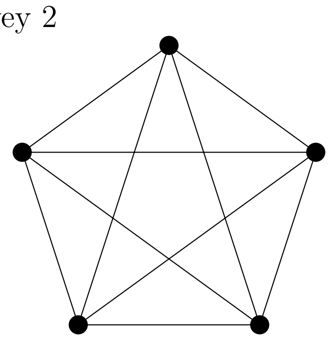
<u>Survey1</u>	<u>Survey2</u>	<u>Survey3</u>
<u>Apple</u> - Orange	<u>Apple</u> - Orange	Apple - <u>Orange</u>
<u>Orange</u> - Mango	<u>Orange</u> - Mango	<u>Orange</u> - Mango
Mango - <u>Banana</u>	<u>Mango</u> - Banana	Mango - <u>Banana</u>
<u>Banana</u> - Pineapple	<u>Banana</u> - Pineapple	<u>Banana</u> - Pineapple
Pineapple - <u>Apple</u>	<u>Pineapple</u> - Apple	<u>Pineapple</u> - Apple
Apple - <u>Banana</u>	<u>Apple</u> - Banana	Apple - <u>Banana</u>
<u>Banana</u> - Orange	<u>Banana</u> - Orange	<u>Banana</u> - Orange
<u>Orange</u> - Pineapple	<u>Orange</u> - Pineapple	<u>Orange</u> - Pineapple
Pineapple - <u>Mango</u>	<u>Pineapple</u> - Mango	<u>Pineapple</u> - Mango
<u>Mango</u> - Apple	<u>Mango</u> - Apple	Mango - <u>Apple</u>

(a) Draw a directed graph for each of the surveys.

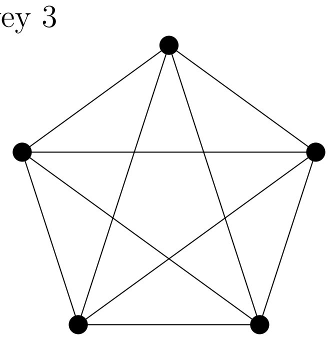
i. Survey 1



ii. Survey 2



iii. Survey 3



continued on next page ⇒

(b) Define a tournament and verify that each of these surveys defines a tournament.

(c) For each survey:

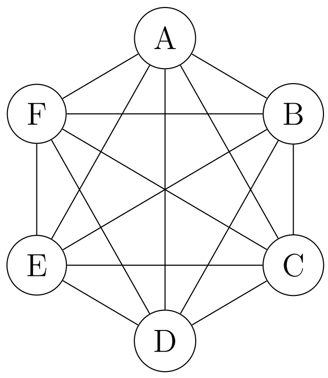
i. What is the score sequence?

ii. Is it transitive?

iii. Is it strongly connected?

(d) For each survey, if possible, find a ranking of the preferences.

[14] 6. Consider the following weighted graph (weights in table):



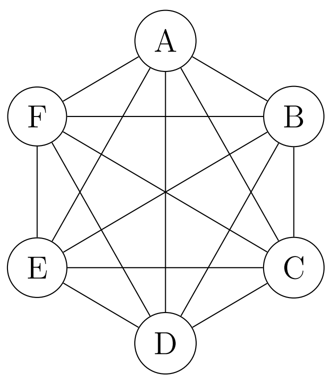
	A	B	C	D	E	F
A	–	3	5	3	6	2
B	3	–	2	4	4	7
C	5	2	–	6	8	8
D	3	4	6	–	2	7
E	6	4	8	2	–	5
F	2	7	8	7	5	–

(a) Find an lower bound for the solution to the travelling salesman problem by removing A.

(b) Find an lower bound for the solution to the travelling salesman problem by removing D.

(c) Which of the bounds found above, in parts (a) and (b), is the better bound?

(The weighted graph again.)



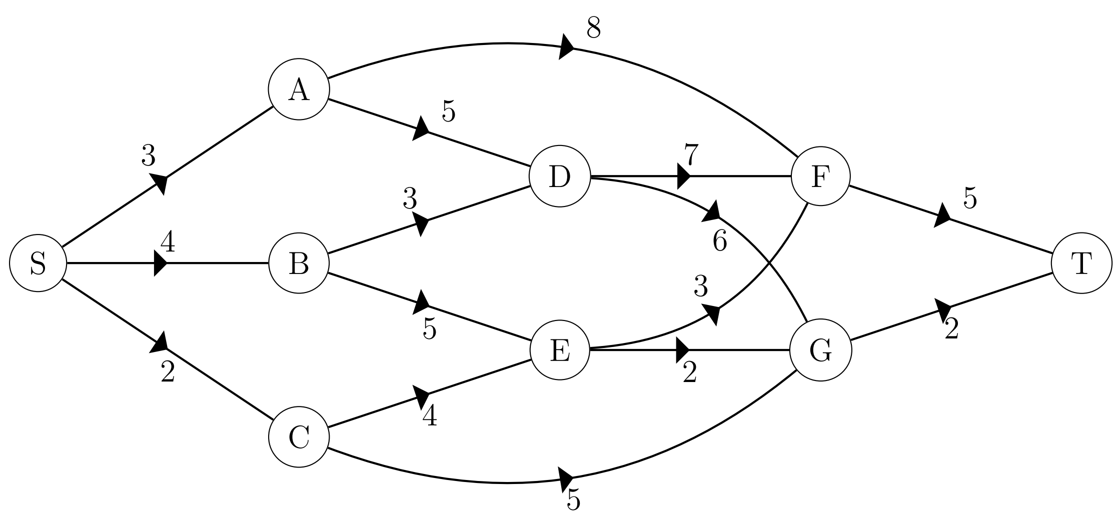
	A	B	C	D	E	F
A	–	3	5	3	6	2
B	3	–	2	4	4	7
C	5	2	–	6	8	8
D	3	4	6	–	2	7
E	6	4	8	2	–	5
F	2	7	8	7	5	–

(d) Find an upper bound for the solution to the travelling salesman problem by starting at A.

(e) Find an upper bound for the solution to the travelling salesman problem by starting at D.

(f) Which of the bounds found above, in parts (d) and (e) is the better bound?

[14] 7. In the following weighted digraph:



(a) Find the longest path from S to T

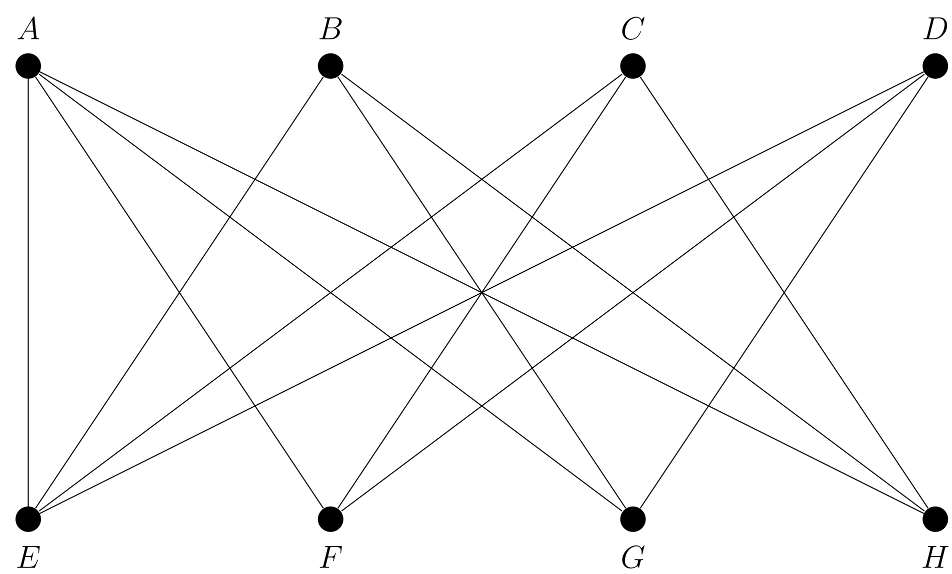
(b) Complete the following table according to scheduling the events represented in the above graph:

E-Earliest start time L-Latest start time F-Float time

	SA	SB	SC	AD	AF	BD	BE	CE	CG	DF	DG	EF	EG	FT	GT
E															
L															
F															

[8] 8. (a) State Kuratowski's Theorem.

(b) Use Kuratowski's Theorem to show that the following graph is not planar.



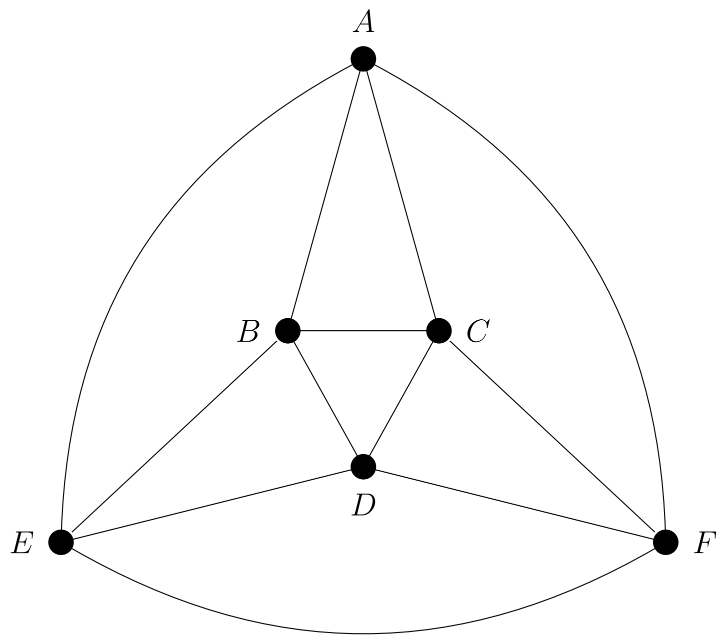
[12] 9. (a) State Euler's formula, and the conditions under which it holds.

(b) Use the above to show that for a simple connected planar graph G having V-vertices, E-edges and no triangles, then $E \leq 2V - 4$.

(c) Show that $K_{3,3}$ is not planar.

(d) Draw an example of a connected planar graph having 6 vertices and 9 edges and no triangles (if one exists). How many faces does it have?

[12] 10. Below is a planar drawing of the octahedron graph G .



(a) Draw the dual, G^* of the octahedron graph G. (label the vertices of G^* with $f_1, f_2, \ldots$)

(b) What is the name of G^*

(c) What is the length of the longest cycle in G^*? Give an example of such a cycle.

(d) List the edges in G which correspond to the edges listed in part (c). What do these edges form?