Definitions, page 33

A **transformation** of the points in the plane is a rearrangement of all the points in the plane.

If no two points are moved to a single position, then we say the transformation is **one-to-one**.

A transformation is **onto** if all the positions in the plane are achieved by some points in the rearrangement.

A **bijection** is a transformation that is both onto and one-to-one.

Definition, page 33

A transformation is **rigid** if it preserves distance. Such transformations are called **symmetries**.

Basic Symmetries

Rotations

Reflections

Translations

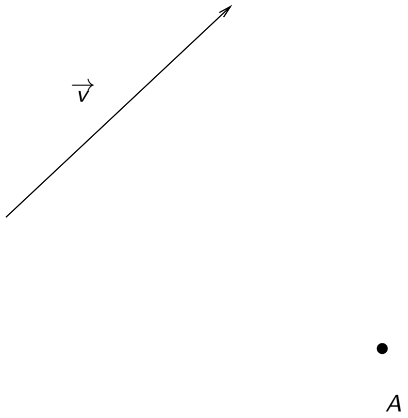
Notation

A rotation is defined by an angle θ and a centre C, and is denoted by $f = rot(C, \theta)$.

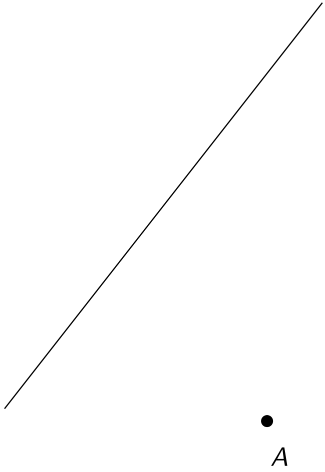
A reflection is defined by a line ℓ and is denoted by $f = refl(\ell)$.

A translation is defined by a vector $\overrightarrow{v}$ and is denoted $f = trans(\overrightarrow{v})$

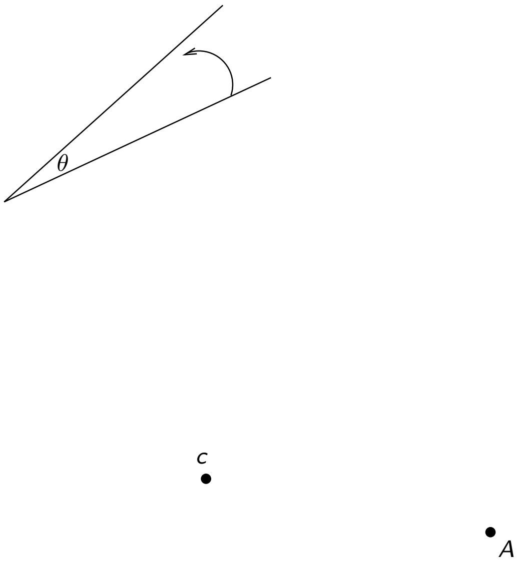
Find the image of A under the symmetry $f = trans(\overrightarrow{v})$



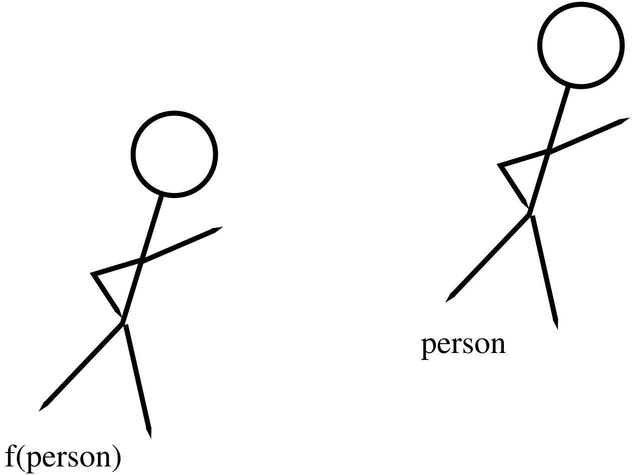
Find the image of A under the symmetry $f = refl(\ell)$



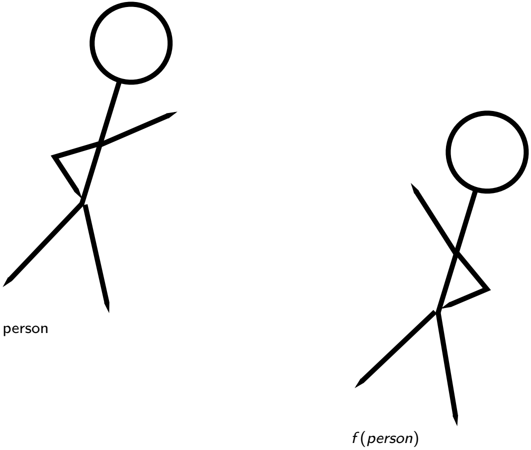
Find the image of A under the symmetry $f = rot(c, \theta)$



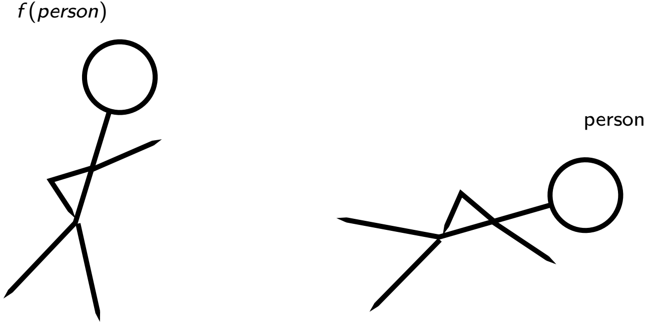
Find $\overrightarrow{v}$ the vector of translation of the symmetry
$f = trans(\overrightarrow{v})$



Find ℓ the line of reflection of the symmetry $f = \mathit{refl}(\ell)$



Find the center and angle of the symmetry $f = rot(c, \theta)$



Compositions of Symmetries

The composition of two symmetries is also a symmetry.

Theorem (The Classification Theorem for Plane Symmetries)

Every symmetry of the plane is either a composition of a translation followed by a rotation, or it is a composition of a translation followed by a reflection.

Find the image of A under the composition of the symmetries $f = refl(\ell)$ followed by $f = rot(c, 60^\circ)$