

Hyperbolic Geometry

Recall Euclid's Fifth Postulate:

For every line ℓ and point P that does not lie on ℓ , there exists a unique line m through P and parallel to ℓ .

There are two ways that can be different, we could replace 'a unique' with 'many' or 'none'.

Hyperbolic Geometry

Hyperbolic Fifth Postulate:

Given a line ℓ and a point P not on ℓ , there are many lines on P parallel to ℓ .

★ - In this context, parallel MEANS the lines do not meet.

Poincaré Model of a Hyperbolic Geometry

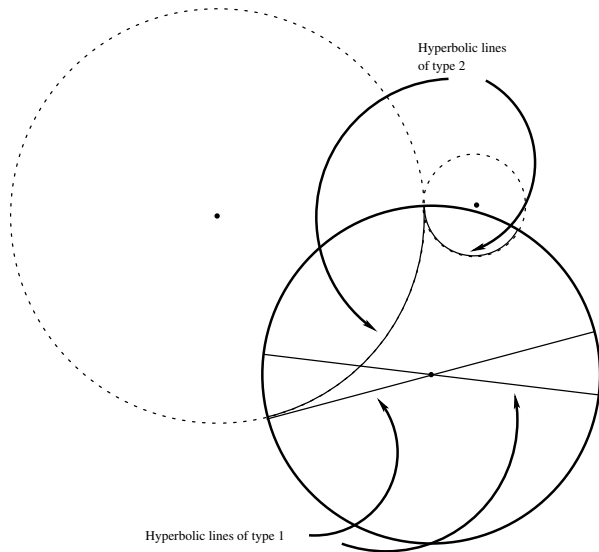
Given H , a circle with center O :

Points of the geometry are all the points that are STRICTLY inside of H . This does not include the points on the circle itself.

Lines of the geometry are of two types:

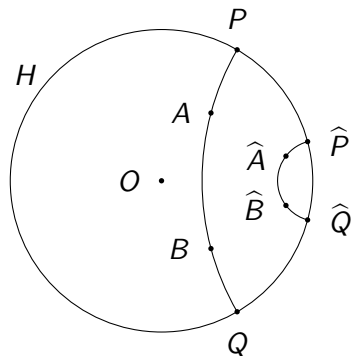
- ▶ diameters
- ▶ Parts of circles that are perpendicular to H

Poincaré Model

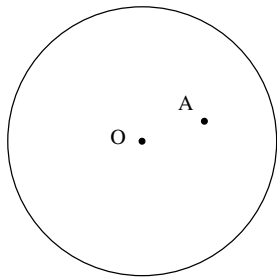


Hyperbolic distance

The hyperbolic distance between two hyperbolic points A and B is determined by a ratio of distances between the points A and B and the points P and Q on the hyperbolic horizon on the unique line joining A and B .

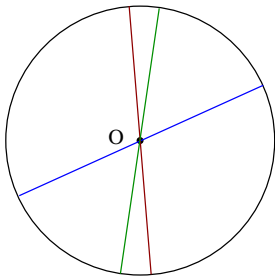


Circle Inversion



Hyperbolic Lines through O , the center of H

All hyperbolic lines that pass through O are diameters.



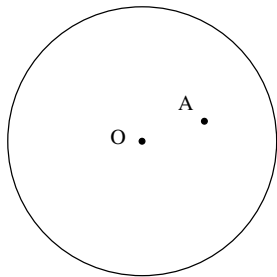
Hyperbolic Lines through A , a point other than the center

The perpendicular bisector of the line $A \operatorname{inv}(A)$ is called the center line for A . We sometimes denote this ℓ_A .

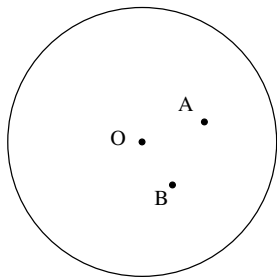
All of the lines through A are:

- ▶ The diameter that passes through A .
- ▶ The part interior to H of a circle that has center on ℓ_A and passes through the point A .

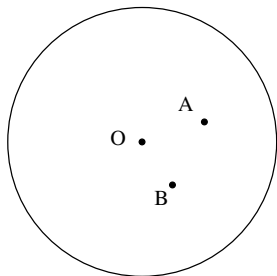
Hyperbolic Lines through A



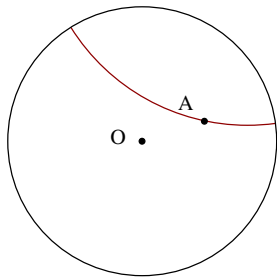
The Hyperbolic line passing through A and B ,
(Construction 2)



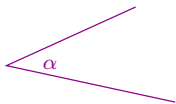
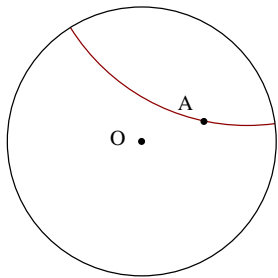
The Hyperbolic line passing through A and B , (Alternate Method)



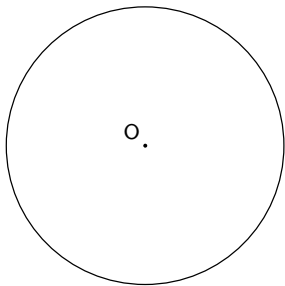
Construction 3; page 161 - Angle of 90°



Construction 3; page 161 - Given an angle



An equilateral triangle



Sum of the angles of a triangle

In a hyperbolic plane, the sum
of the angles of a triangle is
LESS THAN 180° .