MATH 2030 – Combinatorics 1

Worksheet 2

1. Charlene has a collection of beads. She has 21 ornate metal beads, 12 blue beads, 15 yellow beads, and 13 red beads. Each of the metal beads is distinct, however, two beads of the same colour (blue, yellow or red) are indistinct. We refer to the beads that are blue or yellow or red as 'coloured beads'.

   For the following, find the count. Answers should be in exponential or factorial notation.

   (a) How many different strings of 10 beads can be made if only metal beads are used?

   (b) How many different strings of 10 beads can be made if only coloured beads are used?

   (c) Charlene wants to give her friend 10 coloured beads. How many different ways can she do this?

   (d) Charlene wants to give her friend 10 metal beads. How many different ways can she do this?

   (e) Charlene wants to make a string of beads with 10 blue beads and two metal beads. How many different strings of this type can be made?

   (f) Charlene's friend wants a similar bead string, but in red. Charlene gives her 10 red beads and two metal beads. How many different strings can Charlene's friend make?

2. A school teacher with a class of 20 students wants to show the class a film, but he would like them to decide which film they would prefer to see. He has a movie about Koalas (K), a movie about Elephants (E), and a movie about Polar bears (P).

   For the following, find the count. Answers should be in exponential or factorial notation.

   (a) Before he actually asks the class for their preferences, he decides that he will randomly choose one student to speak about their preference for a film. He wants to pick a different student for each film. How many ways can he do this?

   (b) As he questions the class, he writes a letter indicating their preference next to their name. How many different lists could he make?

   (c) Instead of recording the preferences on a list, he decides to just put checks in one of three boxes (the boxes correspond with the films). How many different results could he obtain?

3. Find the probability:

   (a) Five dice are thrown at once. What is the probability that they all show the same value?

   (b) Five dice are thrown at once. What is the probability that at least one of them is a one (1)?

   (c) Seven (7) cards are drawn at random from a standard deck. What is the probability that (at least) one of them is an ace?

   (d) Five (5) cards are drawn at random from a standard deck. What is the probability that there is a pair (two cards with the same number). (Note: for this example, if there is more than one pair, or there is more than two of the same number, we would still consider it as having a pair.)

   (e) A bag contains 12 blue beads, 15 yellow beads and 13 red beads. Three beads are drawn at random. What is the probability that they are all the same colour?

4. For each of the following values, give an example where the count would be the given value:

   (a) $\binom{8}{3}$

   (b) $\frac{8!}{(8-3)!}$

   (c) 8^3

   (d) $\binom{8+3-1}{3-1}$

5. Give a combinatorial proof of the identity:

$$\sum_{i=0}^{n} \binom{n}{i}^2 = \binom{2n}{n}.$$

   Hint: You may use the known identity $\binom{n}{r} = \binom{n}{n-r}$.