MATH 2030 – Combinatorics 1

Worksheet 3

1. For the following, find the count. Answers can be in a variety of forms, including use of the notation $S(n, k)$, Stirling numbers of the second kind.

   (a) Five (5) children show up at your door trick-or-treating. You have 12 different pieces of candy. How many ways could you distribute the candy so that every child gets at least one piece?

   (b) You are volunteering to fill shoeboxes to be sent as donations. You are currently filling 5 shoeboxes (all the shoeboxes are the same). You want to distribute 12 colourful (distinct) pencils into the boxes. How can you distribute the pencils so that all of the shoeboxes get at least one pencil?

   (c) You need to put away some toys. There are 5 distinct storage spots in the house. How many ways can you store 12 different teddy bears?

   (d) You are volunteering to fill shoeboxes to be sent as donations. You are currently filling 5 shoeboxes (all the shoeboxes are the same). You want to distribute 12 identical pencils into the boxes. How can you distribute the pencils so that all of the shoeboxes get at least one pencil?

   (e) You are helping a friend clean up his sons toys. They have brought out 5 indistinguishable storage boxes. How many ways can you put away 12 distinct teddy bears?

   (f) Five (5) children show up at your door trick-or-treating. You have 12 indistinguishable pieces of candy. How many ways could you distribute the candy so that every child gets at least one piece?

   (g) You are helping a friend clean up his sons toys. They have brought out 5 indistinguishable storage boxes. How many ways can you put away 12 identical toy robots?

   (h) You need to put away some toys. There are 5 distinct storage spots in the house. How many ways can you store 12 identical robot toys?

2. Shelley bought a new shoe rack to store her 12 pairs of shoes. This rack has 3 rows, each row can fit 4 pairs of shoes.
   (a) How many distinct ways can she arrange her shoes?
   (b) Suppose all that Shelley cares about is the row that a pair of shoes is on. How many different ways can she arrange her shoes?

3. Using steps of the form Up (U) which steps from $(x, y) \to (x, y+1)$ and Right (R) which steps from $(x, y) \to (x+1, y)$ we form what is called a UR path.
   (a) Find the number of UR paths from (0,0) to (5,4).
   (b) Find the number of UR paths from (0,0) to (7,4).
   (c) Find the number of UR paths from (0,0) to (7,4) where two U steps are never taken in a row.

4. (a) What is the coefficient of x^{11} in $(1+x)^{13}$?
   (b) What is the coefficient of x^8 in $(1+2x)^{11}$?
   (c) What is the coefficient of x^4y^7 in $(x+y)^{11}$?
   (d) What is the coefficient of $x^3y^4z^5$ in $(x+y+z)^{12}$?

5. Suppose an RNA chain was digested and the following was found:
   G fragments: ACG ACG CAUG UCAG
   U,C fragments: C AC AU AG GC GU GAC
   (a) How many different RNA strands have the same base pairs?
   (b) How many different RNA strands have the same G fragments?
   (c) How many different RNA strands have the same U,C fragments?
   (d) How many different RNA strands have the same G fragments and U,C fragments?