MATH 2030 – Combinatorics 1

Worksheet 4

1. (a) If $\sum_{i=0}^{25} \binom{25}{i} 3^i = x^{50}$, what is the value of x?

   (b) If $\sum_{i=0}^{10} \binom{10}{i} 7^i = x^{15}$, what is the value of x?

   (c) If $\sum_{i=0}^{30} \binom{30}{i} 5^i\, 2^{30-i} = x^{15}$, what is the value of x?

2. Show that given any sequence of $mn + 1$ distinct real numbers, there is either a subsequence of length $m + 1$ that is increasing, or a subsequence of length $n + 1$ that is decreasing.

3. Let n be a positive integer $n > 3$. Let $m = \left\lfloor \frac{n+2}{2} \right\rfloor$.

   Suppose $S_n = \{1, 2, \ldots, n\}$, $A \subseteq S_n$ and $|A| > m$; then there are three elements in A such that one is the sum of the other two. (I.E. There is $a_1, a_2, a_3 \in A$ such that $a_1 = a_2 + a_3$.)

4. Let A be any set of 19 distinct integers chosen from the arithmetic progression $1, 4, 7, \ldots, 97, 100$. Prove there must be two distinct integers whose sum is 104.

5. (a) Prove that if 27 distinct positive odd integers, each less that 100, are chosen there is some pair of numbers whose sum is 102.

   (b) What is the smallest number of distinct positive even integers, each less than 100, that would need to be chosen to guarantee there is a pair of numbers whose sum is 102.