Worksheet 5

1. (a) Fictional University has a course in magic that has 120 students in it. Of these, there are 55 students from Otherland, there are 42 students that are trolls, and there are 48 students with purple hair. There are 20 that are trolls from Otherland. There are 17 which are trolls with Purple hair. There are 13 that are from Otherland and have purple hair. There are 5 that are purple haired trolls from Otherland.

   i. How many magic students are not trolls, are not from Otherland and do not have purple hair?

   ii. How many students with purple hair are not trolls and not from Otherland?

   (b) (Adapted from *Discrete and Combinatorial Mathematics - Grimaldi*)
   A group of 65 community works throw a summer party.
   Of these people 21 bring hot dogs, 35 bring fried chicken, 28 bring salad and 32 bring dessert.
   Also 13 bring hot dogs and fried chicken, 10 bring hot dogs and salad, 9 bring hot dogs and dessert, 12 bring fried chicken and salad, 17 bring fried chicken and dessert, 14 bring salads and dessert.
   There are 4 people who bring hot dogs, fried chicken and salad; 6 bring hot dogs, fried chicken and desserts; 5 bring hot dogs, salads and desserts; and 7 bring fried chicken, salads and desserts.
   People who did not bring a food item are responsible for set up and clean up.

   i. How many of the 65 people will be on set up and clean up duty?

   ii. How many are bringing only hot dogs?

   iii. How many are bringing exactly one food item?

2. How many arrangements of $1\,2\,3\,4\,5\,6\,7\,8\,9$ do not have as a subsequence of consecutive entries the sequences $1\,2\,3$ or $4\,5\,6$ or $7\,8\,9$?

3. How many arrangements of $1\,2\,3\,4\,5\,6\,7\,8\,9$ do not have as a subsequence of consecutive entries the sequences $1\,2\,3$ or $3\,4\,5$ or $3\,6\,9$?

4. A bowl of Halloween candy contains 5 KitKats, 5 Areos and 5 Smarties. How many different ways can a child select 10 candies (chocolate bars) from the dish?

5. If 8 standard dice are rolled at once, what is the probability that all 6 numbers appear?

6. How many integers less than (or equal to) 200 have no factors in common with 210? (Hint: $210 = 2 \cdot 3 \cdot 5 \cdot 7$)

7. How many integers less than (or equal to) 200 are prime? (Hint: If a number less than 200 is the product of two numbers neither of which is 1, then one of the numbers must be 2, 3, 5, 7, 11, or 13)

8. How many derangements of $1\,2\,3\,4\,5\,6\,7\,8$ start with $1\,2\,3\,4$ in some order?

9. How many derangements of $1\,2\,3\,4\,5\,6\,7\,8$ start with $5\,6\,7\,8$ in some order?