MATH 2030 – Combinatorics 1

Worksheet 8
Review

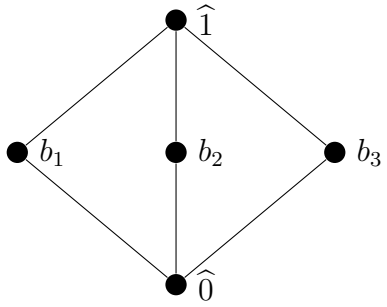
1. Let $X = \{1, 2, 3, 4\}$.
   (a) How many binary relations are there on the set X?
   (b) How many binary relations are there on the set X which are reflexive?
   (c) How many binary relations are there on the set X which are symmetric?
   (d) How many binary relations are there on the set X which are transitive?
   (e) How many binary relations are there on the set X which are complete?
   (f) How many binary relations are there on the set X which are asymmetric and complete?

2. Let $A = \{1, 2, 3, 4\}$ and let X be the set of subsets of A.
   (a) How many binary relations are there on the set X?
   (b) How many binary relations are there on the set X which are reflexive?
   (c) How many binary relations are there on the set X which are symmetric?
   (d) How many binary relations are there on the set X which are complete?
   (e) How many binary relations are there on the set X which are asymmetric and complete?

3. Given a set X where $|X| = n$, which is greater, the number of reflexive binary relations on X or the number of irreflexive binary relations on X?

4. Given a set X where $|X| = n$, which is greater, the number of symmetric binary relations X or the number of antisymmetric binary relations on X?

5. How many different linear extensions can you find for the following partial order? What is the dimension of the partial order?



6. For each of the 3 following partial orders do the following:
   (a) i. Find a longest chain.
   ii. Find the appropriate number of disjoint antichains which contain all the elements.
   (b) i. Find a largest antichain.
   ii. Find the appropriate number of disjoint chains which contain all the elements.
   (c) i. Find the height.
   ii. Find the width.
   iii. Find bounds on the dimension.