Worksheet 6 MATH 2030 Fall 2015

1.

$$S_2 = 12 + 27 + 18 + 13 + 18 + 20$$
$$= 108$$

$$S_3 = 8 + 5 + 7 + 10$$
$$= 30$$

$$S_4 = 3$$

$$e_2 = S_2 - \binom{3}{1} S_3 + \binom{4}{2} S_4$$
$$= 108 - (3)\,30 + (6)\,3$$
$$= 36$$

2. Let a_1 be the property that the subsequence 123 appears

a_2 456

a_3 789

$S_1 = 3 \cdot 7!$
$S_2 = 3 \cdot 5!$
$S_3 = 3!$

$$e_1 = S_1 - \binom{2}{1} S_2 + \binom{3}{2} S_3$$
$$= 3 \cdot 7! - 2 \cdot 3 \cdot 5! + 3 \cdot 3!$$
$$= 3 \cdot 7! - 6 \cdot 5! + 3 \cdot 3!$$

3. Let a_1 be the property that the subsequence 123 appears.
a_2 345
a_3 369

$S_1 = N(a_1) + N(a_2) + N(a_3) = 7! + 7! + 7! = 3 \cdot 7!$
$S_2 = N(a_1a_2) + N(a_1a_3) + N(a_2a_3) = 5! + 5! + 0 = 2 \cdot 5!$
$S_3 = N(a_1a_2a_3) = 0$

$e_1 = S_1 - \binom{2}{1}S_2 + \binom{3}{2}S_3$
$3 \cdot 7! - 2 \cdot 5!$

4. Let a_i be the property that a 1 does not appear.
a_2 2
a_3 3
$\vdots$ $\vdots$
a_6 a_6

Want exactly 4 numbers to appear; so exactly two numbers do not appear.

$S_2 = \binom{6}{2}4^8$
$S_3 = \binom{6}{3}3^8$
$S_4 = \binom{6}{4}2^8$
$S_5 = \binom{6}{5}1^8$
$\{S_6 = \binom{6}{6}0^8 = 0\}$

$e_2 = S_2 - \binom{3}{1}S_3 + \binom{4}{2}S_4 - \binom{5}{3}S_5 + \binom{6}{4}S_4$
$e_2 = \binom{6}{2}4^8 - \binom{3}{1}\binom{6}{3}3^8 + \binom{4}{2}\binom{6}{4}2^8 - \binom{5}{3}\binom{6}{5}1^8 + 0$

Hence prob $= \frac{e_2}{6^8} = \frac{\binom{6}{2}4^8 - \binom{3}{1}\binom{6}{3}3^8 + \binom{4}{2}\binom{6}{4}2^8 - \binom{5}{3}\binom{6}{5}1^8}{6^8}$

5 Let a_1 be the property that the number is divisible by 2
a_2 3
a_3 5
a_4 7
a_5 11
a_6 13

From last weeks worksheet we have:

$S_2 = 132$, $S_3 = 24$, $S_4 = S_5 = S_6 = 0$

So $e_2 = S_2 - \binom{3}{1} S_3 + \binom{4}{2} S_4 - \binom{5}{3} S_5 + \binom{6}{4} S_6$

$= 132 - 3(24)$

$= 60$

Calculations for S_2:

$\left\lfloor \frac{200}{6} \right\rfloor = 33$ $\left\lfloor \frac{200}{10} \right\rfloor = 20$ $\left\lfloor \frac{200}{14} \right\rfloor = 14$ $\left\lfloor \frac{200}{22} \right\rfloor = 9$ $\left\lfloor \frac{200}{26} \right\rfloor = 7$

$\left\lfloor \frac{200}{15} \right\rfloor = 13$ $\left\lfloor \frac{200}{21} \right\rfloor = 9$ $\left\lfloor \frac{200}{33} \right\rfloor = 6$ $\left\lfloor \frac{200}{39} \right\rfloor = 5$ $\left\lfloor \frac{200}{35} \right\rfloor = 5$

$\left\lfloor \frac{200}{55} \right\rfloor = 3$ $\left\lfloor \frac{200}{65} \right\rfloor = 3$ $\left\lfloor \frac{200}{77} \right\rfloor = 2$ $\left\lfloor \frac{200}{91} \right\rfloor = 2$ $\left\lfloor \frac{200}{143} \right\rfloor = 1$

Calculations for S_3 (zeros are omitted):

$\left\lfloor \frac{200}{30} \right\rfloor = 6$ $\left\lfloor \frac{200}{42} \right\rfloor = 4$ $\left\lfloor \frac{200}{66} \right\rfloor = 3$ $\left\lfloor \frac{200}{78} \right\rfloor = 2$ $\left\lfloor \frac{200}{70} \right\rfloor = 2$ $\left\lfloor \frac{200}{110} \right\rfloor = 1$

$\left\lfloor \frac{200}{130} \right\rfloor = 1$ $\left\lfloor \frac{200}{154} \right\rfloor = 1$ $\left\lfloor \frac{200}{182} \right\rfloor = 1$ $\left\lfloor \frac{200}{105} \right\rfloor = 1$ $\left\lfloor \frac{200}{165} \right\rfloor = 1$ $\left\lfloor \frac{200}{195} \right\rfloor = 1$