MATH 1210 Assignment #1

Due: January 22, 2016; At the start of class

Reminder: all assignments *must* be accompanied by a signed copy of the honesty declaration available on the course website.

1. Use mathematical induction to prove

   $1 + 5 + 9 + 13 + \cdots + (4n + 1) = (n + 1)(2n + 1)$ for all $n \geq 1$.

2. Use mathematical induction to prove

   $2 + 5 + 8 + 11 + \cdots + (9n - 1) = \dfrac{3n(9n + 1)}{2}$ for all $n \geq 1$.

3. Use mathematical induction to prove

   $n + (n + 1) + (n + 2) + (n + 3) + \cdots + (5n) = 3n(4n + 1)$ for all $n \geq 1$.

4. Use mathematical induction to prove

   $1^2 + 2^2 + 3^2 + 4^2 + \cdots + (2n)^2 = \dfrac{n(2n + 1)(4n + 1)}{3}$ for all $n \geq 1$.

5. Use mathematical induction to prove

   $3^{3n} - 1$ is divisible by 13 for all $n \geq 1$.

6. Write each of the following using sigma notation:
   (a) $1 + 3 + 5 + 7 + \cdots + 111$
   (b) $\frac{5}{12} + \frac{6}{14} + \frac{7}{16} + \frac{8}{18} + \cdots + \frac{49}{100}$
   (c) $2 + 6 + 10 + 14 + \cdots + (8n + 6)$

7. Using the known formulas, evaluate each of the following:
   (a) $\sum_{j=1}^{12}(j + 1)^2$
   (b) $\sum_{j=6}^{20} 4j - 7$
   (c) $\sum_{j=6}^{20}(j - 5)(j + 3)$