MATH 1210 Assignment #2

Due: February 3, 2016; At the start of class

Reminder: all assignments *must* be accompanied by a signed copy of the honesty declaration available on the course website.

1. Simplify and express the complex numbers in Cartesian form

   (a) $\overline{\left(\frac{(6-2i)^4}{(1+3i)^4}\right)}$

   (b) $\frac{(i-1)^3}{(i+1)^2}$

   (c) $\left(\frac{i}{e^{i\pi}}\right)^{25}$

2. Simplify and express the complex numbers in polar and exponential forms using the principal value of the argument θ, $\theta \in (-\pi, \pi]$

   (a) $\left(\overline{\sqrt{3}+3i}\right)^2$

   (b) $-\frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$

   (c) $(-12+i)^3(-12-i)^3$

3. Find all solutions of the equation

   $x^6 + x^3 + 1 = 0.$

4. Find all solutions of the equation

   $z^8 = -1.$

5. Let z_1 and z_2 be 2 complex numbers. Show that

   $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}.$

6. Let z be a complex number. Using mathematical induction prove that

   $\overline{z^n} = \overline{z}^n$, for all $n \geq 1$.

7. Consider the following polynomial $P(x) = x^5 - 2x^4 + 4x^3 + 2x^2 - 5x$.

   (a) Verify that $1 + 2i$ is a root of $P(x) = 0$.

   (b) Find all the roots of $P(x) = 0$.

   (c) Factor $P(x)$ into the product of real linear and irreducible real quadratic factors.

8. (a) Show that $(x - i)$ and $(x - 1)$ are linear factors of

   $$x^4 - 2(1+i)x^3 + 4ix^2 + 2(1-i)x - 1 = 0.$$

   (b) Factor the polynomial $x^4 - 2(1+i)x^3 + 4ix^2 + 2(1-i)x - 1$ in linear factors.

9. Consider the following polynomial

   $$P(x) = x^5 - 11x^4 + 43x^3 - 73x^2 + 56x - 16.$$

   (a) Show that $P(x)$ can be rewritten as $P(x) = Q(x)(x-4)$ and $P(x) = T(x)(x-1)$ where $Q(x)$ and $T(x)$ are polynomials in x. Give the degree of $Q(x)$ and $T(x)$.

   (b) Show that 4 is a root of multiplicity 2 of $P(x)$.

   (c) Factor $P(x)$.