MATH 1210 Assignment #3

Due: February 22, 2016; At the start of class

Reminder: all assignments *must* be accompanied by a signed copy of the honesty declaration available on the course website.

1. Consider the polynomial
   $P(x) = \sum_{k=0}^{2015} \frac{(-1)^k}{k+1} x^k$.
   (a) Show that $P(x)$ must have at least one positive real root.
   (b) Show that $P(x)$ has no negative real roots.
   (c) Show that if z is any root of $P(x)$, then $|z| < 2020$.

2. Consider the polynomial $P(x) = x^3 + 4x^2 + k^3 x + 3$, where k is some integer. Find all possible values of k such that $P(x)$ has a rational root. (Clearly explain why there are no other values of k that work.)

3. In each part of this question: (i) use Descartes rules of signs to state the number of possible positive and negative zeros of the polynomial; (ii) use the bounds theorem to find bounds for zeros of the polynomial; (iii) use the rational root theorem to list all possible rational zeros of the polynomial; (iv) use this information to find all the zeros of the polynomial.
   (a) $6x^5 + 7x^4 - 13x^3 - 85x^2 - 50x$
   (b) $x^9 + 3x^8 + 3x^7 + 3x^6 + 6x^5 + 6x^4 + 4x^3 + 6x^2 + 6x + 2$

4. Let $A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 2 & 2 & 0 & 2 \\ 2 & 0 & 1 & 6 \end{bmatrix}$; $B = (b_{ij})_{3\times 4}$, $b_{ij} = i - j$.
   Find a matrix X such that $3(X^T + I) = 2(B^T A)^T$, or explain why such X does not exist.

5. Let x and y be real numbers; $A = \begin{bmatrix} x & y \\ 0 & -x \end{bmatrix}$.
   Prove that for any integer $n \geq 0$, $A^{2n+1} = \begin{bmatrix} x^{2n+1} & x^{2n}y \\ 0 & -x^{2n+1} \end{bmatrix}$.

6. Let $\mathbf{u}$ be a vector from point $(1, -4, 0)$ to point $(-2, 3, 5)$; v be the vector with length 5 in the opposite direction to $\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$.
   (a) Find $\mathbf{2u} \times \mathbf{v} + (\mathbf{u} \cdot \mathbf{v})|\mathbf{v}|\hat{\mathbf{u}}$, where $\hat{\mathbf{u}}$ is the unit vector in the direction of $\mathbf{u}$.
   (b) Find a vector of length 8 perpendicular to both **3u+v** and **u-2v**.

7. Let $\mathbf{u}$ and $\mathbf{v}$ be two unit vectors such that $\mathbf{u} \cdot \mathbf{v} = \frac{1}{32}$.
(a) Prove that vectors **u-v** and **3u+3v** are perpendicular.
(b) Find the angle between vectors **2u+6v** and **3u-v**.
Hint: Consider how dot product of a vector with itself is related to its length.