MATH 1210 Assignment #4

Due: March 16, 2016; At the start of class

Reminder: all assignments *must* be accompanied by a signed copy of the honesty declaration available on the course website.

1. Find the line through $(3, 1, -2)$ that intersects and is perpendicular to the line $x = -1+t$, $y = -2 + t$, $z = -1 + t$ with $t \in \mathbb{R}$.

2. Find an equation for the plane containing the two lines of equations

$$\langle x, y, z\rangle = \langle 0, 1, 2\rangle + t_1\langle 2, 3, -1\rangle, \quad t_1 \in \mathbb{R}$$

and

$$\langle x, y, z\rangle = \langle 2, -1, 0\rangle + t_2\langle 4, 6, -2\rangle, \quad t_2 \in \mathbb{R}.$$

3. Show that the lines $x - 3 = 4t_1$, $y - 4 = t_1$, $z - 1 = 0$ and $x + 1 = 12t_2$, $y - 7 = 6t_2$, $z - 5 = 3t_2$ with $t_1, t_2 \in \mathbb{R}$ intersect, and find the point of intersection.

4. Solve the linear system

$$\mathbf{AX} = \mathbf{B}$$

where $\mathbf{B}^T = [0, 0, 0]$ and

$$\mathbf{A} = \begin{bmatrix} 1 & -2 & 0 & 3 \\ 2 & -3 & -1 & -4 \\ 3 & -5 & -1 & -1 \end{bmatrix}$$

Specify the method used to solve the linear system.

5. Compute the determinant of

(a) $\mathbf{A} = \begin{bmatrix} 1 & -2 & 0 & 3 \\ 2 & -3 & -1 & -4 \\ 3 & -5 & -1 & -1 \end{bmatrix}$

(b) $\mathbf{A} = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 2 & 1 & -3 & 4 \\ 0 & 2 & -2 & 3 \\ 1 & 1 & -4 & -2 \end{bmatrix}$

(c) $\mathbf{A}^5$ when $\mathbf{A}$ is defined as in (b).

(d) $-\mathbf{A}$ when $\mathbf{A}$ is defined as in (b).

(e) $\mathbf{A}^T$ when $\mathbf{A}$ is defined as in (b).

6. Find all values of λ for which $\det(\mathbf{A}) = 0$ for $\mathbf{A} = \begin{bmatrix} \lambda - 4 & 0 & 0 \\ 0 & \lambda & 2 \\ 0 & 3 & \lambda - 1 \end{bmatrix}$.

7. Consider the following linear system

$$\begin{aligned} 2x - y + 2z &= 2, \\ x - y - z &= 1, \\ 4x + 2y - z &= 0. \end{aligned}$$

(a) What is the rank of the augmented matrix of the linear system?

(b) What is the rank of the coefficient matrix of the linear system?

(c) By using Gauss-Jordan elimination, solve the linear system.

8. Consider the following linear system

$$\begin{aligned} 2x_1 + 2x_2 - x_3 + x_5 &= 0, \\ -x_1 - x_2 + 2x_3 - 3x_4 + x_5 &= 0, \\ x_1 + x_2 - 2x_3 - x_5 &= 0, \\ x_3 + x_4 + x_5 &= 0. \end{aligned}$$

(a) By using Gauss-Jordan elimination, solve the linear system.

(b) Write your solution(s) using the basic solution(s).

(c) Is the trivial solution a solution of this linear system? Explain your answer.